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THREE ESSAYS ON SHORT-TERM MACROECONOMICS: BUSINESS FLUCTUATIONS, LARGE DEVALUATIONS AND INFLATION DYNAMICS

THESE

Présentée à l'Ecole des Hautes Etudes Commerciales
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par

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Pour l'obtention du grade de
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Je dédie cette thèse à mon frère.

TABLE OF CONTENTS

Introduction

At least since the Great Depression, explaining why there are business fluctuations has been one of the biggest challenges that the science of economics has had to face. The hope is that if we could better understand recessions, then we could also be more successful in overcoming them. This dissertation consists of three papers that are part of the general endeavor of economists to understand these fluctuations.

The first paper discusses, for a particular model, whether a result related to fluctuations would still hold if time were modeled as continuous rather than discrete. The two other papers focus on price stickiness. The second paper discusses why, after a large devaluation, prices of non-tradables may change by only a small amount in comparison to the magnitude of the devaluation. The third paper examines price adjustment in a model in which information is imperfect and it is costly to change prices.

The Random-Lags Approach: Application to a Microfounded Model

Aghion et al. (2004), ABB from now on, present an open-economy model that explains why economies at an intermediate level of financial development may be more unstable than either more or less developed economies. It has been objected that ABB's result holds only if time is modeled as discrete and vanishes if time is modeled as continuous. The intuition is as follows. In ABB's model the economy jumps back and forth over a steady state. If time were modeled as continuous, however, then the economy would go through the steady state; and once it was at the steady state, it would be stuck there and fluctuations would not occur. I use the approach of Invernizzi and Medio (1991), IM from now on, to show that ABB's result still holds when the discrete-time assumption is relaxed.

IM consider that discrete time and continuous time are the two extremes between which there is a continuum of intermediate cases. They recast this time-modeling issue into a heterogeneity issue. IM's insight is that at the macro level, the assumption that production takes place in discrete time implies in fact two assumptions: production at the firm level must occur at discrete intervals and production of all firms must be synchronized. IM accept the lag assumption at the micro level, which is often realistic, but reject the synchronization assumption, which is usually unrealistic. In order to get a model that is not synchronized, they assume that lags are heterogeneous and random. They show that there is a correspondence between random lags following a degenerate distribution of variance zero and the discrete-time model, whereas the continuous-time model corresponds to the case in which random lags follow an exponential distribution.

I show that ABB's result (an economy at an intermediate level of development may be more unstable than either more or less developed economies) is still valid for many intermediate cases between the discrete-time model and the continuous-time model.

Small Price Change Response to a Large Devaluation in a Menu-Cost Model

After a devaluation, one could expect that the price of imported goods expressed in domestic currency would increase, leading to an increase in the prices of non-tradables. In an empirical paper based on five large devaluation episodes (in Argentina, Brazil, Korea, Mexico and Thailand), however, Burstein et al. (2005a) find a very slow adjustment in the prices of non-tradable goods and services after large devaluations. Burstein et al. (2005b) develop a model that can account for this phenomenon. They conclude by noting a shortcoming of their paper: in their model the prices of non-tradables do not change at all, while in reality these prices do change, albeit by far less than the exchange rate does.

I consider an alternative, simpler model and explore under which conditions moderate menu costs can explain the muted response of the prices of non-tradables. The key new element in this alternative model is a nominal friction in wage-setting (generated by menu costs for changing wages). I find for example that although my model is based on menu costs, it is able to deliver not only an absence of response of the prices of non-tradables, but also a small response. This shows that partial adjustment is compatible with menu costs being the sole nominal rigidity, even in the case of large devaluations. I also discuss the existence of multiple equilibria and the role of the central bank's credibility.

Can a Hybrid Sticky-Price and Sticky-Information Model Reconcile Stylized Facts on the Frequency of Individual Price Changes and on Inflation Dynamics?

Theories based on imperfect information rather than on menu costs are making a comeback. But explaining price stickiness solely on the basis of imperfect information can be criticized on several counts. One such criticism is that this implies that every firm changes its prices every quarter (except in the special and unrealistic case of zero steady-state inflation), whereas this does not match empirical facts on the frequency of individual price changes. In this paper I build a hybrid model combining the standard model based on menu costs with the sticky-information model of Mankiw and Reis (2002) based on imperfect information. Because of menu costs, not all firms will change their prices each period in the hybrid model. However, the question arises as to whether the gain in this microeconomic dimension is obtained at the cost of a loss in the macroeconomic dimension. I show that this need not be the case. Focusing on an economic environment compatible with the literature and for which computation is particularly simple, I show that the gain in the microeconomic dimension does not necessarily imply a loss in the macroeconomic dimension.

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Part I

The Random-Lags Approach: Application to a Microfounded Model*

Abstract

It is well known that a one-dimensional discrete-time model may yield endogenous fluctuations while this is impossible in a one-dimensional continuous-time model. Invernizzi and Medio (1991) recast this time-modeling issue into an aggregation issue. They have proposed a "random-lags approach" as a way of preserving fluctuations while relaxing the discrete-time assumption. The present paper applies this approach to the model of Aghion, Bacchetta and Banerjee (2000), and shows that their result that economies at an intermediate level of financial development may be prone to economic fluctuations continues to hold when the discrete-time assumption is relaxed.

Keywords: continuous time, discrete time, fluctuations, aggregation.

JEL Classification Number: E32.

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1 Introduction

One explanation of economic fluctuations is based on financial frictions. Bernanke and Gertler (1989) have shown that borrowing constraints on firms can amplify and increase the persistence of temporary shocks. Kiyotaki and Moore (1997), Aghion, Banerjee, and Piketty (1999) and Azariadis and Smith (1998) have shown that these constraints can lead to oscillations in the context of a closed economy. Aghion, Bacchetta, Banerjee (2004), ABB from now on, study the case of a small open economy.

The goal of ABB's paper is to explain why economies at an intermediate level of development may be more unstable than either more or less developed economies. They propose a model in which fluctuations are more persistent for intermediate values of the borrowing constraint (which correspond to an intermediate level of financial development)[1]. In order to derive their result, ABB assume time to be discrete. The problem is that there is no reason (other than technical simplicity) to make this assumption. The present paper shows that their result still holds when the discrete-time assumption is relaxed.

In order to prove ABB's result while relaxing the discrete-time assumption, I use the approach of Invernizzi and Medio (1991), IM from now on. They recast this time-modeling issue into an aggregation issue. IM's insight is that at the macro level the assumption that production takes place in discrete time implies in fact two assumptions: production at the firm level must occur at discrete intervals and production of all firms must be synchronized.[2] If firms are not synchronized, then at any given date some firms are finishing their production; in this case, aggregate production might best be seen as continuous although production is a discrete-time variable at the agent level. IM accept the lag assumption at the micro level, which is often realistic, but reject the synchronization assumption, which is usually unrealistic. In order to build a model that is not synchronized, they assume that lags are heterogeneous and random. Thus, the date of production of different firms cannot

[1] They also show that in economies at an intermediate level of financial development full capital account liberalization may destabilize the economy (while foreign direct investment does not destabilize it). But I will focus here on their first result.

[2] IM's approach is general and applies to any discrete-time model of the form $X_t = f(X_{t-1})$. In specific applications, the terminology "lags" may sometimes seem inappropriate. For example, in the production case, this lag is the exogenously-given time-interval between two production processes, which may include periods that one may not want to call "lags", such as the duration required to produce. But for simplicity I will stick to the lag terminology.

be synchronized, since their lags are different. IM show that their model converges toward the discrete-time model when the dispersion of lags tends toward zero. Then they show that if the dispersion of lags is small enough, the endogenous fluctuations of the discrete-time model are preserved.[3]

The present paper applies this approach to ABB's paper and shows not only that fluctuations are preserved, but also that the point of the ABB model (fluctuations are greater for economies at an intermediate level of financial development) holds while relaxing the discrete-time assumption. The plan of the paper is as follows: after presenting ABB's Model (§2), I apply IM's approach to it (§3) and present concluding remarks (§4).

2 A specific one-dimensional, discrete-time example: ABB's model

The goal of ABB's paper is to explain why economies at an intermediate level of financial development may be more unstable than either more or less developed economies. I focus here on the simplest version of ABB's model. It features a small open economy with two types of agents: entrepreneurs and owners of a local input. Entrepreneurs produce a tradable good which is both a consumption and a capital good. The price of this tradable good is taken as given because of the small open economy assumption. The other input in the production of the tradable good is a local input that is not owned by entrepreneurs. Entrepreneurs can borrow at an interest rate $r-1$, which is exogenous, given the assumption of a small open economy. Entrepreneurs, however, may not be able to borrow as much as they wish because they are subject to a borrowing constraint. This borrowing constraint takes the form of a constant credit-multiplier μ. Entrepreneurs can borrow up to μ times their wealth. The parameter μ captures the level of financial development. When $\mu = 0$ entrepreneurs cannot borrow, whereas when $\mu = \infty$ there is no limit to the amount entrepreneurs can borrow.

At time t, after consumption, entrepreneurs have wealth W_t at their disposal. Because of the borrowing constraint they can borrow up to μW_t. If they choose to borrow the maximum amount possible, they will have $(1+\mu)W_t$ at their disposal. They buy the quantity z_t of local input at price p_t, and use the difference $K_t = (1+\mu)W_t - p_t z_t$ as a tradable input. They

[3] In fact IM do not only show that fluctuations still yield: they are mainly interested in the chaotic properties of these fluctuations.

choose z_t in such a way as to maximize their own production. Production is a function $y(K_t, z_t)$ of the tradable and local inputs. In their basic example, ABB assume that the production function is a Leontief: $y = min(\frac{K_t}{a}, z_t)$. Entrepreneurs receive an exogenous income e and at the end of the period repay the principal with interest $r\mu W_t$ to the lender. Then, entrepreneurs consume a fraction α of their wealth (this behavior can be derived from log utility).

The equilibrium price p_t adjusts to set z_t equal to the supply of local input assumed to be a constant z. If $z > \frac{K}{a}$ (this happens when W_t is so small that current investment cannot absorb the total supply of the non-tradable input), then there is excess supply of the non-tradable input and thus its price is null. If $z = \frac{K}{a}$ then it can be shown that $p_t = \frac{(1+\mu)W_t - az}{z}$. The case $z < \frac{K}{a}$ cannot exist because it cannot be optimal for the entrepreneurs to choose a quantity of the costly tradable input in excess of what is useful given the amount of local input.

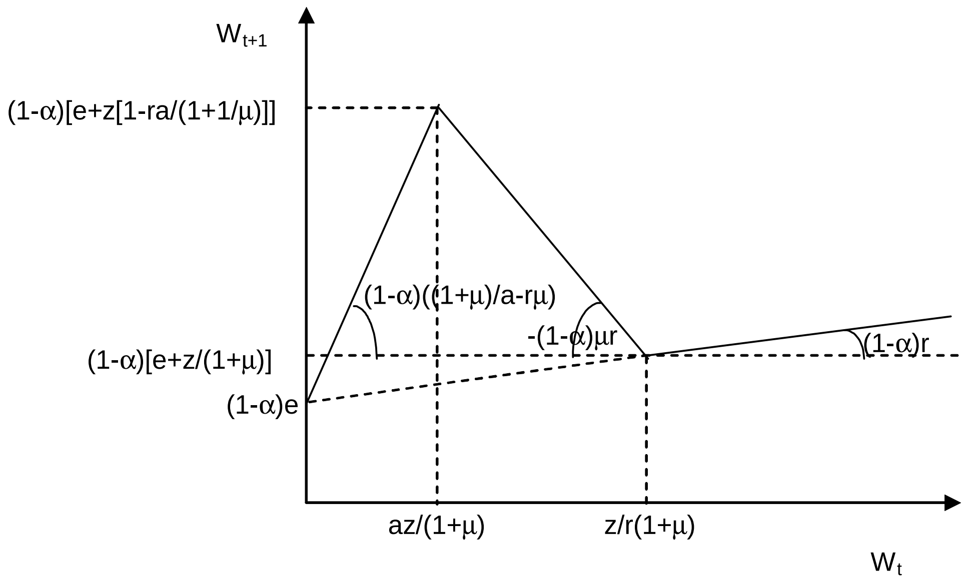
Entrepreneurs can also choose not to borrow the maximum amount possible (they are then not constrained). In this case the return on their investment is $r - 1$, and their wealth in the next period is $W_{t+1} = (1-\alpha)(e + rW_t)$.

The dynamics $W_{t+1} = f(W_t)$ of the entrepreneurs' wealth are therefore given by:

$$W_{t+1} = (1-\alpha)\left\{e + \max\left[\min\left[\left(\frac{1+\mu}{a} - r\mu\right)W_t, z - r\mu W_t\right], rW_t)\right]\right\} .$$

Assuming $e > 0$, $1 > ar$, and $(1-\alpha)r < 1$, these dynamics are represented graphically in Figure 1.

Figure 1: Dynamics of the entrepreneurs' wealth



The steady state is given by the intersection between this curve and the diagonal. There are fluctuations only if the curve has a negative slope at the steady state, i.e. if the intersection is on the second segment (and these fluctuations are permanent only if the slope is a negative number lower than -1). It can be shown that the steady state will be on the first segment if μ is small enough, and on the third segment if it is large enough. Thus, fluctuations occur (the steady state can be on the second segment) only for an intermediate level of financial development (i.e. for intermediate values of μ).

ABB explain the basic mechanism underlying their model as follows. It is a combination of two forces: on one side, greater investment leads to greater output and ceteris paribus, higher profits. Higher profits improve creditworthiness and fuel borrowing that leads to greater investment. Capital flows into the country to finance this boom. At the same time, the boom in investment increases the demand for the country-specific factor and raises its price relative to the output good. This rise in input prices leads to lower profits and therefore, reduced creditworthiness, less borrowing and less investment, and a fall in aggregate output. Of course, once investment falls all these forces get reversed and eventually initiate another boom. The reason why an intermediate level of financial development is important for this result is easy to comprehend: at very high levels of financial development, most firms' investment is not constrained by cash flow so shocks to cash flow are irrelevant. On the other hand, at very low levels of financial development, firms cannot borrow very much in any case and therefore their response to cash-flow shocks will be rather muted.

If there are fluctuations, one of the two forces described above should dominate sometimes and the other one should dominate at other times. But in between there should be a point at which the two forces cancel each other out. This point would be a steady state. In a single-variable, continuous-time model governed by a differential equation of degree 1,[4] the economy would be stuck at this steady state and would not fluctuate after all. But ABB assume that time is discrete. In this case the economy may overshoot the steady state, and then jump back over the steady state and be ready for a new cycle. It is this discrete-time assumption that I will try to relax.

3 Extension of ABB's model with random lags

I first discuss IM's random-lags approach (§3.1) on which my extension of the ABB model is based, then this extension is presented (§3.2).

3.1 IM's random-lags approach

Consider any variable X and assume that its dynamics in discrete time are given by:

$$X_t = f(X_{t-1}) \ . \tag{1}$$

For example, it may be useful to think of X as representing aggregate production finished at time t.[5] The lag is the time required to produce (a new cycle of production starts right after the preceding is finished). The discrete-time dynamics equation (1) says that aggregate production finished at time t is a function of aggregate production finished at time $t-1$.

[4] A single-variable, continuous-time model governed by a differential equation of degree n can be regarded as a n-variable, continuous-time model governed by n differential equations of degree 1.

[5] When applying this approach to ABB I will choose X=Wealth of the entrepreneurs.

Instead of the single representative firm implied in equation (1), one may consider an economy consisting of a large number of firms differing only by their production lags. Assume that this lag is random, and the density function ψ gives its distribution. Then equation (1) can be written as:

$$X_t = \int_0^\infty \psi(s) f(X_{t-s}) ds \ . \tag{2}$$

Equation (2) indicates that aggregate production finished at time t is the sum of production processes started in the past. Aggregate production carried out s periods ago, X_{t-s}, generates total production $f(X_{t-s})$. Only a fraction $\psi(s)$ of this production will, however, be finished at time t. Thus the production process beginning at time $t-s$ will contribute $\psi(s)f(X_{t-s})$ to aggregate production at time t. Notice that if $\psi(s) = 0$ for $s \neq 1$ then lags are not random anymore, and equation (2) can be simplified to $X_t = f(X_{t-1})$. The strength of the approach proposed by IM is to keep the discrete-time assumption at the micro level, a realistic assumption, but to dismiss the assumption of perfect synchronization, which is usually unrealistic.

Assuming that $\psi(s)$ is a gamma density

$$\psi(s) = \frac{1}{(n-1)!} n^n s^{n-1} e^{-ns} \ , \tag{3}$$

with expectation 1 and variance $\frac{1}{n}$ (where n$\geq$1; the economic interpretation of this parameter is presented below), IM show that equation (2) is equivalent to the following differential equation:

$$\left(\frac{1}{n}D + 1\right)^n X = f(X) \ , \tag{4}$$

where $D = \frac{d}{dt}$ is the time-derivative operator.

Here the parameter n plays a crucial role. If n is infinite, then the variance of the distribution of lags is zero, and equation (4) describes a discrete-time model.[6] If $n = 1$, then equation (4) describes a single-variable, continuous-time model governed by a differential equation of degree 1. For intermediate values of n, equation (4) describes an intermediate case between discrete time and first-order continuous time.

n can be interpreted as the number of successive and independent elementary operations needed to complete production, the duration of each

[6] It can be shown that the differential equation (4) tends toward (1) when n tends toward infinity.

elementary operation being random and following an exponential distribution. For comparability, only production processes are considered for which the whole production process is expected to last one period. If there are n operations, then each operation is assumed to have an expected duration of $\frac{1}{n}$.[7] When n rises, the expected production lag stays the same (1 by construction), but the dispersion around this expected value decreases. The reason is that when there are many operations, it is very unlikely that operations are always short or always long. Thus by the law of large numbers the time gained on short operations tends to be canceled by the delay of some other, long operations. At the limit as $n \to \infty$ the distribution of lags is degenerate and one obtains the discrete-time model.

$n = 1$ corresponds to the continuous-time model: in this case equation (4) is a differential equation of degree 1. $n = 1$ is the opposite of $n = \infty$ (as the continuous-time model is the opposite of the discrete-time model) because the distribution of production duration for $n = 1$ is the opposite of the distribution of production duration in the discrete-time model in the following sense: the distribution for $n = 1$ has the property that production duration can take any positive value (instead of only one as in the discrete-time model) and that the probability of a firm finishing production in the next infinitesimal interval of time is completely independent of the time that has elapsed since production last occurred (instead of being completely determined by the time that has elapsed since production last occurred as in the discrete-time model).

Values of n between 1 and ∞ correspond to intermediate cases between continuous time and discrete time. IM show that permanent fluctuations that appear in discrete time still remain in intermediate cases close enough to discrete time. Intuitively, if n is large enough, then the standard deviation of production duration is small enough, and the tendency of production of various firms to get out of synchronization is weak enough, such that permanent fluctuations arising in the discrete-time model are not canceled out. Remember that fluctuations arise in the discrete-time model because all entrepreneurs can borrow large amounts when they start with large wealth, putting upward pressure on the price of the non-tradable input, leaving them with small profits and thus small wealth for the next period. This whole process collapses if production of various firms are sufficiently out of synchronization.

[7] Then it can be shown that the production duration will follow the gamma distribution given by equation (3).

Formally IM show[8] that the condition for having a periodical solution is

$$0 > f'(\overline{X}) = -\frac{1}{\cos^n(\frac{\pi}{n})}\ , \tag{5}$$

where $\overline{X}$ is the steady state of X defined by: $\overline{X} = f(\overline{X})$.

It is easy to derive equation (5) by taking the following linear approximation of equation (4) around the steady state (using $f(X) \approx f(\overline{X}) + \left(X - \overline{X}\right) f'(\overline{X})$):

$$\left[\left(\frac{1}{n}D + 1\right)^n - f'(\overline{X})\right]\left(X - \overline{X}\right) = 0\ . \tag{6}$$

The eigenvalues λ are given by the solutions of $\left(\frac{1}{n}\lambda + 1\right)^n = f'(\overline{X})$. Notice that for $f'(\overline{X}) < 0$ the eigenvalues with the higher real component are a complex number (with the imaginary component different from zero) and its complex conjugate. Their real component is $n\left\{\left[\left|f'(\overline{X})\right|\right]^{\frac{1}{n}}\cos(\frac{\pi}{n}) - 1\right\}$.

For $n = 1$, this maximal real component is equal to $-\left|f'(\overline{X})\right| - 1$, which is negative. Thus all real components are negative and the system is stable.

For $n = 2$, this maximal real component is equal to -1 (except if $\left|f'(\overline{X})\right| = \infty$), which is negative. Thus all real components are negative and the system is stable.

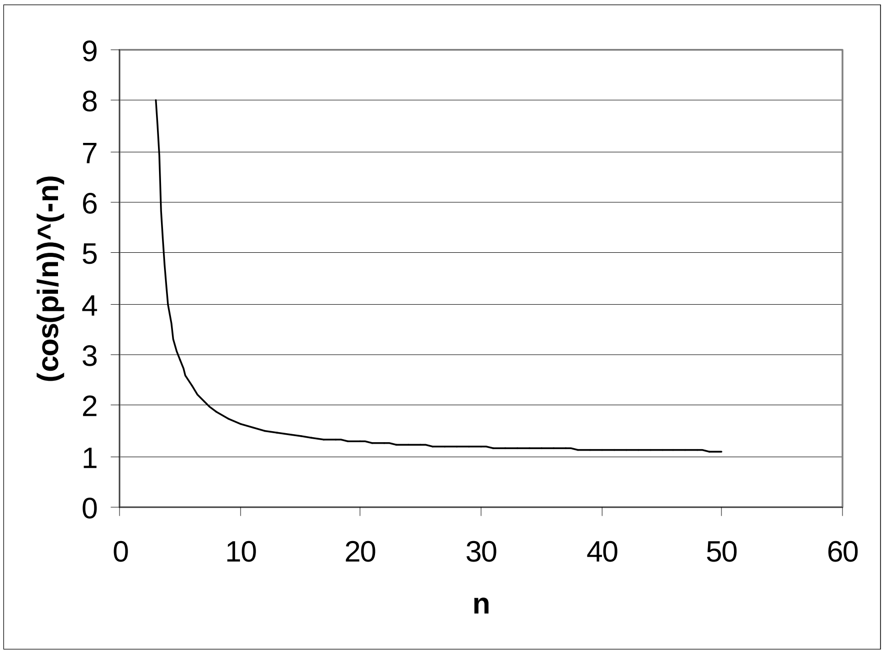
For $n > 2$, the real component of all eigenvalues is negative and the system is stable if $f'(\overline{X}) > -\frac{1}{\cos^n(\frac{\pi}{n})}$, whereas there is at least one dimension in which the system is unstable if $f'(\overline{X}) < -\frac{1}{\cos^n(\frac{\pi}{n})}$. If $f'(\overline{X}) = -\frac{1}{\cos^n(\frac{\pi}{n})}$, it can be shown that there is a periodical solution.

If $n \to \infty$, then $-\frac{1}{\cos^n(\frac{\pi}{n})} \to -1$ and, as usual in discrete-time models, there are permanent fluctuations if the slope of f at the steady state is smaller than -1.

The following graph (which is plotted for $n > 2$) shows that $\frac{1}{\cos^n(\frac{\pi}{n})}$ is already close to its horizontal asymptote for small n. This is an indication that fluctuations will still yield even for fairly small n.

[8]They don't explicitly write this equation, but it is a straightforward implication of their paper.

Figure 2: Dependence of the domain of fluctuations on the lag distribution



3.2 Robustness of ABB's results

I now show that qualitatively ABB's result is still valid for intermediate cases close enough to a discrete-time model.

Using $X \equiv W$ in equation (4), the dynamics are given by

$$\left(\frac{1}{n}D+1\right)^n W = f(W) \ ,$$

where

$$f(W) = (1-\alpha)\left\{e + \max\left[\min\left[\left(\frac{1+\mu}{a} - r\mu\right)W, z - r\mu W\right], rW\right)\right]\right\} \ .$$

How do the properties of the steady state depend on μ? First the steady state $\overline{W}$ must be computed. The steady state satisfies the following equation $\left(\frac{1}{n}D+1\right)^n \overline{W} = f(\overline{W})$, which, since $\overline{W}$ is constant, reduces to $\overline{W} = f(\overline{W})$. Thus, the steady state is the same as in ABB's discrete-time case. Assuming

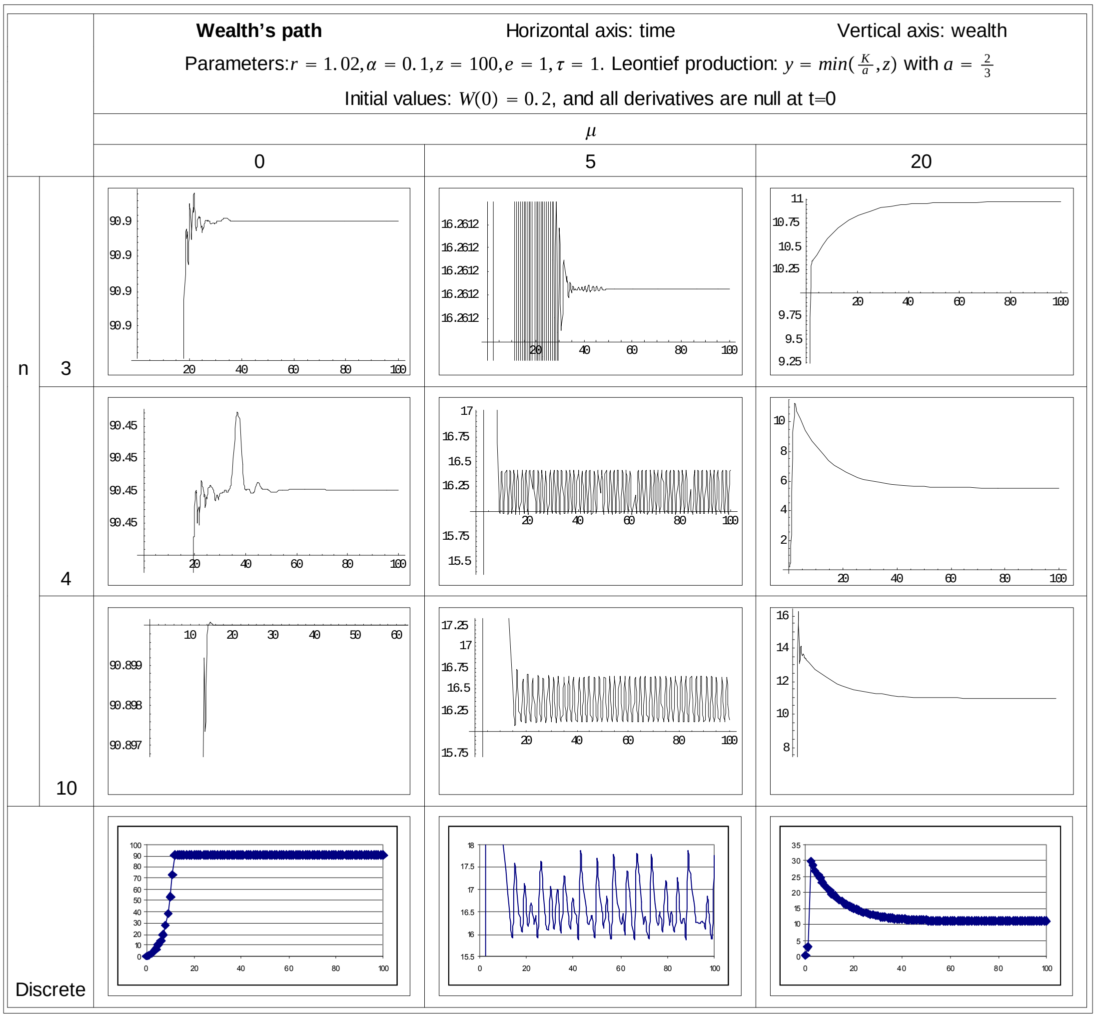
a is big enough, the steady state will be either on the second or the third segment. Let's discuss the stability of the steady state. Linearizing around the steady state yields:

$$\left[\left(\frac{1}{n}D+1\right)^n - f'(\overline{W})\right]\left(W-\overline{W}\right)=0 \ .$$

The eigenvalues λ are given by the solution of $\left(\frac{1}{n}\lambda+1\right)^n = f'(\overline{W})$. If the steady state is on the third segment, then $0 < f'(\overline{W}) < 1$ and all eigenvalues have negative real components. Thus the steady state is stable and there will be no permanent fluctuations. If the steady state is on the second segment, then $f'(\overline{W})$ is negative, and there will be permanent fluctuations if $f'(\overline{W})$ is sufficiently negative. The difference with respect to the discrete case is that "sufficiently negative" no longer means that $f'(\overline{W}) < -1$, but that $f'(\overline{W}) < -\frac{1}{\cos^n(\frac{\pi}{n})}$. Thus as long as n>2 the difference from the discrete-time model is quantitative (how negative $f'(\overline{W})$ needs to be in order to get permanent fluctuations) rather than qualitative.

The previous argument shows that for $n > 2$ ABB's result is still valid (although, as I will show, the set of values of e for which it holds becomes more restrictive). To illustrate, I now compute the wealth path for several values of n and μ with given parameter values. Figure 3 shows the relevant graphs.

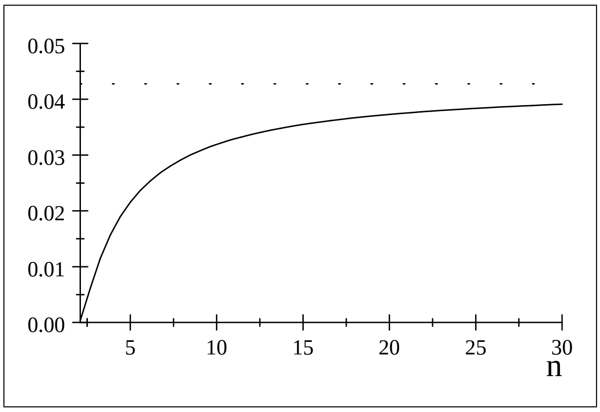
Figure 3: Wealth path for various levels of financial development and lag distribution



The last row of Figure 3 shows that when n is infinite (i.e. the discrete-time case) fluctuations are permanent for an intermediate value of μ (here $\mu = 5$), while the system converges quickly if μ is smaller ($\mu = 0$) or larger ($\mu = 20$). When $n = 10$ or even $n = 4$ (which is fairly close to first-order continuous time: the standard deviation of the lags is still half what it would be in the first-order continuous case), it is still true that the fluctuations are permanent for intermediate values of μ only. But this is no longer true for $n = 3$: even for the intermediate value $\mu = 5$ there are no permanent fluctuations. It is easy to understand why fluctuations cannot be permanent for $n = 3$, given our parameters' values. Simple algebra shows that for $n > 2$, if $\frac{e}{z} < \frac{1-(1-\alpha)r}{\frac{1}{\cos^n(\frac{\pi}{n})}+(1-\alpha)r}$ there will always be permanent fluctuations for some value of μ: there will always be a μ such that the steady state is on the second segment and the negative slope is steep enough for fluctuations to be permanent. With our parametrization, however, there will be no such μ for $n = 3$ since this inequality is not satisfied ($\frac{e}{z} = \frac{1}{100}$ and $\frac{1-(1-\alpha)r}{\frac{1}{\cos^3(\frac{\pi}{3})}+(1-\alpha)r} = 9.1 \times 10^{-3}$). Changing the value of e would change the results. If e were small enough there would be permanent fluctuations for intermediate values of μ also for $n = 3$. On the other hand, for any n we could choose a value e high enough such that there are no permanent fluctuations.

Compared to the similar condition prevailing in the discrete-time model $\frac{e}{z} < \frac{1-(1-\alpha)r}{1+(1-\alpha)r}$, the condition $\frac{e}{z} < \frac{1-(1-\alpha)r}{\frac{1}{\cos^n(\frac{\pi}{n})}+(1-\alpha)r}$ becomes more restrictive when n gets smaller (that is, when we move away from the discrete-time case). ABB's result that permanent fluctuations occur for intermediate values of μ is true only for a particular set of values for the parameters (for example, e must be small enough). As n decreases this set shrinks. But as long as $n > 2$, this set is never empty. In this sense the result ABB obtain in discrete time is still qualitatively valid for any $n > 2$, but quantitatively the set shrinks. How fast? Figure 4 features $\frac{1-(1-\alpha)r}{\frac{1}{\cos^n(\frac{\pi}{n})}+(1-\alpha)r}$ as a function of n for $\alpha = 0.1$ and $r = 1.02$ (the horizontal line is the value for n infinite).

Figure 4: Dependence of the domain of validity of ABB's result on the lag distribution



In the discrete-time model ABB's result holds when $\frac{e}{z} < 0.043$, whereas for $n = 5$ it holds when $\frac{e}{z} < 0.022$. Thus, among the values of parameter

e for which ABB's property holds in the discrete-time model, for more than half of them this property already holds for $n = 5$.

The following intuition explains why the lower bound of e values for which endogenous fluctuations cannot occur (whatever the value of μ) is an increasing function of n. Remember that endogenous fluctuations occur because of cash-flow shocks to firms' capacity to borrow. For endogenous fluctuations to occur, two conditions must be satisfied. First, μ has to be large enough for borrowing to be substantial. Second, μ has to be small enough for firms to be financially constrained. When n gets larger, the tendency of firms to get out of synchronization diminishes, and a smaller μ will suffice to generate enough borrowing for endogenous fluctuations to occur. With smaller μ, the second condition will also be easier to satisfy: firms will still be financially constrained even if their exogenous endowment e is a bit larger. Thus, it is easier to get endogenous fluctuations when n is larger: more pairs (μ,e) are compatible with endogenous fluctuations.

4 Conclusion

Applying Invernizzi and Medio's approach, the present paper has shown that Aghion, Bacchetta and Banerjee's explanation of why economies at an intermediate level of financial development may be more unstable than either more or less financially developed economies is fairly robust to the continuous–time versus discrete-time choice. When the discrete-time assumption is dropped in favor of a random-lags assumption that is an intermediate case between discrete and continuous time, the argument stays qualitatively the same except in extreme cases when the variance of the lags is large (larger than half the variance corresponding to the first-order, continuous-time model).

Possible directions for further research would be to apply the random-lags approach to models other than ABB's model, or to examine whether it can be applied to issues purely related to aggregation rather than to time-modeling.

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Part II

Small Price Change Response to a Large Devaluation in a Menu-Cost Model*

Abstract

In an empirical paper based on five large devaluation episodes in Argentina, Brazil, Korea, Mexico and Thailand, Burstein and al. (2005a) find a very slow adjustment in the prices of non-tradable goods and services after large devaluations. Burnstein and al. (2005b) develop a quantitative general-equilibrium model that can account for this phenomenon. I consider an alternative, simpler model and explore under which conditions moderate menu costs can explain the muted response of the prices of non-tradables. The key new element in this alternative model is a nominal friction in wage-setting (generated by menu costs for changing wages). I find, for example, that although my model is based on menu costs, it is able to deliver not only constant prices of non-tradables, but also small price changes (in reality these prices do change, albeit by far less than the exchange rate). I also discuss the existence of multiple equilibria and the role of central-bank credibility.

Keywords: large devaluation, exchange rate, pass-through, sticky prices, sticky wages.

JEL Classification Number: F31.

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1 Introduction

The impact of monetary policy depends on whether exchange-rate changes induce a change in the terms of trade (i.e. the relative price of home imports in terms of home exports) and of the price of imports relative to non-tradables. Consider, for example, a monetary expansion leading to a currency depreciation. If prices are sticky in producer currency, then the depreciation implies an increase in the price of imports relative to domestic goods. This will lead to expenditure-switching in favor of domestic goods, but the import price increase may feed into domestic inflation. On the other hand, if prices are sticky at the retail level in the consumer currency, then the exchange-rate change per se will have no impact on prices paid by domestic consumers and there will be no imported inflation (the monetary expansion underlying the exchange-rate change may, however, have an impact on domestic consumption through other channels).

The recent empirical literature on international pricing indicates that the reality is between these two extremes: at the dock, prices are sticky partially in producer currency and partially in buyer currency,[1] whereas at the retail level prices are usually sticky in consumer currency (the implication is that there is no expenditure-switching on the consumer side, whereas there might be some from importing firms; the implication for inflation is that higher import prices are not very dangerous as long as they do not translate into higher retail prices that will in turn lead to wage increases and fuel an inflationary spiral[2]).

This conclusion, hinging on the distinction between the dock and retail levels, reconcile the following otherwise diverging findings. Some of the evidence supports consumer-currency pricing. The fact that nominal exchange rates fluctuate widely in comparison to relative price levels (consumer price indexes) indicates that at the retail level, prices are sticky in the consumer currency. Engel (1993) and Engel (1999) present empirical findings that indicate that prices of non-tradables and tradables move to the same extent after

[1] Campa and Goldberg (2005) compute the pass-through in the short run (defined as one quarter) at the level of an aggregated import bundle. They find a lot of heterogeneity across the OECD countries, some pass-throughs being well above 50 percent, and others substantially below.

[2] There are still other implications. For example, the importing firms will have to provide a buffer for the exchange rate shock by reducing their margins, which will reduce their ability to invest if they are credit-constrained. More generally, this kind of effect is magnified if firms have borrowed in foreign currency. See for example Aghion et al. (2004).

a nominal exchange-rate shock. The following evidence on import and export prices supports producer-currency stickiness. Burstein and al. (2005c) argue that tradables and non-tradables do not in fact react in the same way to exchange-rate fluctuations, and that if they seem to do so it is because what is called "tradables" at the retail level incorporate a lot of non-tradable services such as distribution and marketing: the relative price of pure traded goods at the dock to non-tradables does indeed fluctuate after a nominal exchange-rate shock. Obstfeld and Rogoff (2000) find that there is a positive relationship between the nominal exchange rate and the terms of trade: if the domestic currency depreciates then the price of home imports increases relative to home exports. This result is more consistent with producer-currency pricing than with buyer-currency pricing.

These issues raise many theoretical questions concerning the pricing behavior of firms,[3] their expenditure-switching following a nominal exchange-rate shock,[4] the optimal monetary policies and their links with firms' strategies.[5] In these models, price- or wage-stickiness is often modeled as time-dependent pricing (for example, prices are set one period in advance or are set before the exchange-rate shock). This stickiness might be motivated by menu costs, but how high the menu cost has to be for prices to stay sticky[6] is usually not computed. One exception is Burnstein and al. (2005b), henceforth BER.[7]

[3] See for example Bacchetta and Wincoop (2005), Devereux and Engel (2001) and Corsetti and Pesenti (2002). Two papers undertake to explain why consumer prices respond less than import prices to nominal exchange-rate shocks: Bacchetta and Wincoop (2003) model the optimal pricing strategies when importing firms assemble the intermediate imported goods and sell the final good facing competition from domestic non-tradables, whereas the explanation of Burnstein et al. (2005b) is based on the weight of non-tradables (taking distribution services into account).

[4] Obstfeld (2003b) models the expenditure-switching effect of exchange-rate changes that operates at the firm rather than at the consumer level.

[5] See for example: i) Devereux and Engel (2006), concluding that in practice optimal policy might seek to limit exchange-rate fluctuations, ii) Obstfeld (2003a) who on the contrary defends the role of exchange-rate flexibility in international adjustments, iii) Corsetti and Pesenti (2002) who analyze endogenous optimal monetary unions, iv) Bacchetta and Wincoop (2000) study the impact of the exchange-rate system on trade and welfare.

[6] The assumption that at a given date some price-setters cannot choose whether they want to change their prices or not might be more questionable in an open economy than in a closed economy since the nominal exchange rates tend to fluctuate widely.

[7] There are still other exceptions such as Devereux (2006) who explores the relationship between exchange-rate policy and price flexibility, or Landry (2005) who introduces elements of state-dependent pricing and strategic complementarity into a new open economy macroeconomic model with producer-currency pricing.

In a preceding empirical paper based on five large devaluation episodes in Argentina, Brazil, Korea, Mexico and Thailand, Burstein and al. (2005a) find a very slow adjustment in the prices of non-tradable goods and services to large devaluations. BER address the question of why the rate of inflation for non-tradable goods is so much lower than the rate of devaluation. They develop a quantitative general-equilibrium model that can account for this phenomenon. They assume menu costs for changing a price and show that producers of non-tradables might prefer not to change their price at all even if the devaluation is large. There are also cases in which it is not sustainable as an equilibrium phenomenon for firms in the non-tradables sector not to change their prices at all (in these cases it is argued that real shocks are the primary driver of real exchange-rate movements). They incorporate several assumptions into their model that mute the response of the price of non-tradables to the exchange-rate shock. First, the share of tradable goods in the consumer price index (CPI) is small. Second, there are domestic distribution costs associated with the sale of traded goods. Third, there is a low elasticity of the demand for exports. Fourth, there is a moderate elasticity of substitution between tradables and non-tradables.[8] Moreover, they deviate from the Dixit-Stiglitz model, adopting Kimball's (1995) assumption that the elasticity of demand for the output of a monopolistic producer is increasing in its price relative to the prices of its competitors' goods. They conclude, however, by noting a shortcoming of their paper: the price of non-tradables does not change at all, while in reality these prices do change, albeit by far less than the exchange rate.[9]

Like BER, I aim at explaining why the rate of inflation for non-tradable goods is so much lower than the rate of devaluation. I consider an alternative, simpler model and explore under which conditions moderate menu

[8] The price of tradables will change after an exchange-rate shock. The direct impact of this price change on non-tradables will, however, be small since BER assume a moderate elasticity of substitution between tradables and non-tradables. But even if this elasticity were zero, there would still be other channels through which price adjusments could be induced. For example households would ask for higher wages in order to mute the impact of the increase of the prices of tradables on their real wages. This would incite firms to increase their prices in order to pass the price increase of the labor input on to consumers. However, incorporating several assumptions that mute responses allow BER to get sticky prices with moderate menu-costs.

[9] BER focus on rationalizing an equilibrium in which non-tradable goods prices do not change at all. They do not say if their model could yield an equilibrium in which prices do change a little. My conjecture is that it can not (at least if all firms have the same menu costs): I expect that reducing incentives to change the prices of non-tradables in BER's model would not lead to smaller price changes but might only determine whether prices adjust perfectly or not at all.

costs can explain the muted response of the prices of non-tradables. The key new element in this alternative model is a nominal friction in wage-setting (generated by menu costs for changing wages).[10] For tractability, I consider a partial-equilibrium model rather than a general-equilibrium model like that of BER.

The intuition as to why this may explain small (but possibly not zero) changes in the prices of non-tradables is the following. In a setting in which the markup is a constant proportion of the marginal cost, the desired price varies in the same proportion as the marginal cost. The marginal cost can vary through two channels: a productivity change (due to a change in the quantity produced if returns to scale are not constant) or a change in the prices of production factors. A devaluation increases the price of imported goods and tends to move consumption toward non-tradables, thus increasing the quantity of non-tradables produced and reducing marginal productivity (assuming decreasing economies of scale). Since I use the moderate elasticity of substitution between tradables and non-tradables assumed by BER, this first channel by itself motivates a price change, albeit by far less than the exchange rate. This leaves the second channel: the wage (I assume that labor is the only production factor). If workers do not want to reset their wage, this second channel is not active. Then the desired price change of non-tradables firms is small, and this small change will occur if their menu cost is small enough. But why would workers not reset their wage after a large devaluation although they have two incentives to do so: i) to preserve their real wage and ii) to compensate for higher labor disutility due to the increased quantity of labor they must furnish? These two incentives may be so weak that they do not outweigh even a moderate menu cost of resetting wages. First, the change in price level is moderate since the change of the price of non-tradables is small and the share of pure tradables (exclusive of distribution costs) in the CPI is assumed, as in BER, to be moderate. Second, the change of labor is also moderate since, as discussed above, production of non-tradables does not increase much (and the production of tradables doesn't change much either, since the prices of tradables are assumed to adjust completely and there is no substitution between domestic and foreign tradables[11]).

I will try to avoid the shortcoming of BER consisting in not explaining small positive changes in the prices of non-tradables. The difference between

[10] I owe the idea of introducing wage stickiness, and, more generally, the model I use here, to Philippe Bacchetta and Olivier Jeanne.

[11] For simplicity's sake, rather than modeling domestic tradables production, an exogenous endowment of tradables is assumed.

a small price response and no price response may seem to be irrelevant. It is not. A small difference may matter a great deal if it casts doubt upon the underlying theory. In our case, one could think that a simple menu-cost model can explain that prices do not change at all, if the menu cost is high enough, but would not be able to explain (without an exogenous price-staggering process) why prices adjust only partially. If a firm pays the menu cost, why would it not adjust fully? This paper shows that a menu-cost model can explain partial adjustment. In another setting, the same concern has been expressed, for example, by Midrigan (2006) (he proposes an extension of the state-dependent model in order to explain small price changes and other micro-economic facts): "The large number of small price changes observed in the micro-price data might lead one to conclude against state-dependent pricing models."

Assuming staggered price setting (like the Calvo process) and strategic complementarity in price setting is the standard way to generate partial adjustment. However, assuming a time-dependent process is particularly debatable after a large shock. Moreover, whereas the Calvo process is motivated by menu costs, these menu costs do not appear explicitly. Since I want to examine how high these menu costs need to be to explain incomplete price adjustment after a large devaluation, I need to explicitly have these costs in the model.

For realistic values of the parameters, I get strategic complementarity in price and wage setting. If all firms and workers adjust their prices and wages, then any agent choosing to deviate would bear a large cost. Thus, the equilibrium at which all agents adjust always exists for realistic menu cost values. There may however also be other equilibria. If no agent adjusts, then no agent would gain much by adjusting. Since this gain can be wiped out by a small menu cost, no adjustment will also be an equilibrium as long as the menu cost is not too small.

Interestingly, there are still other equilibria. In particular, workers may prefer not to change their wages at all after a transitory devaluation. In this case, firms in the non-tradables sector will not choose to fully adjust their prices to the devaluation. I compute the minimal menu cost for wages and the maximal menu cost for prices such that the price of non-tradables increases by a small amount. The existence of this equilibrium, and more generally the discussion of multiple equilibria, are the main contributions of this paper, whereas BER focus on rationalizing an equilibrium in which non-tradable goods' prices do not change at all. At the core of my paper are figures that make it possible to understand how the set of multiple equilibria depends on values of menu costs.

I also discuss the role of central-bank credibility. A credible central bank can eliminate the equilibrium in which all agents adjust. If the central bank is not credible, it will have to generate a recession to achieve this result.

The plan of this paper is as follows. The assumptions of the model are presented in section 2. Section 3 presents the equilibrium equations. Section 4 shows that small menu costs are enough to prevent a large change in the non-tradables price and that it is possible for the prices of non-tradables to change by a small amount. Section 5 shows that wage rigidity plays a crucial role in getting this result. Section 6 discusses the importance of central-bank credibility. Section 7 presents concluding remarks.

2 Assumptions of the model

This is a small open economy model. Non-tradables are produced with labor, and there is a domestic endowment of tradables. The non-tradable goods market, as well as the labor market, are cleared. Producers of non-tradables are price setters and households are wage setters. The timing is as follows: firms and households predetermine nominal prices before the occurrence of a devaluation shock. First, domestic producers set their prices and households set their nominal wages. Then the state of the world (devaluation or no-devaluation) is revealed, and price and wage setters decide to maintain prices and wages at the preset levels, or to pay the menu cost and change them in response to the shock.

Firms maximize profit. There are two sectors: the tradables and the non-tradables sectors. There is a continuum of differentiated non-tradable goods produced by a mass 1 of monopolistic producers $i \in [0, 1]$ with the production function $y_i = A_N L_i^{\alpha}$.

The country exports or imports a tradable good (balanced trade account is not assumed) for which the law of one price applies. Normalizing the dollar price of this good to 1, the domestic currency price of the good P_T is equal to the exchange rate S:

$$P_T = S \ .$$

For simplicity's sake, the country is assumed to receive an exogenous endowment of tradable goods.

There is a continuum of mass 1 of atomistic households indexed by h. Each household provides its particular brand of labor, and the labor used in production is a CES composite of the different brands given by

$$L_i = \left(\int l_{h,i}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} , \quad \eta > 1 ,$$

where $l_{h,i}$ is the amount of labor provided by household h to firm i. The total amount of this labor composite in this economy is given by $L = \int_0^1 L_i di$.

Households maximize the following utility

$$u_h = c_h - \gamma \frac{l_h^{1+\theta}}{1+\theta}$$

under the budget constraint $c_h P = w_h l_h$, where $P = \left[\nu P_T^{1-\rho} + (1-\nu) P_N^{1-\rho} \right]^{\frac{1}{1-\rho}}$ is the general price level, w_h is the wage received by household h, and c_h is a CES index of the consumption of tradable and non-tradable goods

$$C = \left[\nu^{\frac{1}{\rho}} C_T^{\frac{\rho-1}{\rho}} + (1-\nu)^{\frac{1}{\rho}} C_N^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} , \qquad \rho > 0 .$$

Notice that profit revenues are not included in the budget constraint. This will simplify the expression for the household's opportunity cost of not adjusting its wage after an exchange-rate shock. This assumes that workers have only labor income, while non-tradables producers earn profits but no labor income and do not consume any non-tradables.

The consumption of non-tradable goods is itself a CES composite of different varieties:

$$C_N = \left(\int C_{N,i}^{\frac{\mu-1}{\mu}} di \right)^{\frac{\mu}{\mu-1}} , \qquad \mu > 1 .$$

The structure of nominal stickiness is as follows. In the non-tradables sector, nominal prices are set before the occurrence of the shock, and can be changed after this occurrence at a certain cost to the price-setter (all firms have the same menu cost): if a firm chooses to adjust its prices, then menu costs are subtracted from its profits. Prices can be changed at no cost the next period. This is the same assumption, as for example, in Fishman and Simhon (2005). Their interpretation is that firms receive new inventories in odd-numbered periods, at which time labels must be applied to newly-arrived units. Therefore, in an even-numbered period, changing a unit's price relative to the preceding period involves the additional cost of changing labels; in odd-numbered periods, in contrast, price labels must be applied anyway so a price change is costless. Thus, prices are assumed to be sticky for at most one period. In the same way, wages are set by the households for one period before the occurrence of the shock and can be changed after this occurrence at a certain cost (the same menu cost for all households): if a household chooses to adjust its wages, then menu costs are subtracted from its utility. Alternatively, the exchange-rate shock can be assumed to be transitory and to last (and be expected to last) only one period.

After prices have been set, the economy can be in one of two states characterized by different nominal demands and exchange rates. The state of no-devaluation occurs with a probability that, for simplicity's sake, is assumed to be very small, so that the dependence of the preset levels on what would happen in case of a shock can be disregarded although anticipations are rational. In the no-devaluation state, the exchange rate is given by $S = S_f$ and domestic nominal demand is given by $P_{N,f}C_{N,f} + P_{T,f}C_{T,f} = N_f$. In the devaluation state this becomes $S = S_d$ and $P_{N,d}C_{N,d} + P_{T,d}C_{T,d} = N_d$, where $\frac{S_d - S_f}{S_f}$ is the rate of devaluation. N_f and N_d are exogenous. Notice that $P_N C_N + P_T C_T = N$ can also be written $C * P = N$. There are two ways to interpret the exogeneity of N. First, one could assume that a transaction technology determines the relation between aggregate spending and real money balances: $C = \frac{M}{P}$, where M is the nominal money stock. Then N is simply equal to M chosen by the central bank.[12] A second interpretation of the exogeneity of N is that it is a way to capture other shocks. Whatever the interpretation, C is given by $\frac{N}{P}$ where N is exogenously given. This exogeneity explains why the impact on demand of interests paid or received from the rest of the world need not be considered.

3 Equilibrium equations

The equilibria are given by the six following equations.[13] Each variable in this system of equations is a ratio of the corresponding variable in case of a shock to this variable in the absence of a shock (labelled by the name of the

[12] The choice of M would also have an impact on the exchange rate. If we want to keep the exchange rate shock exogenous in this interpretation, we need to assume that there is a (transitory) disconnect between the exchange rate and M.

[13] See proof in appendix.

corresponding variable with an index "r").[14]

3.1 Optimization by firms in the non-tradables sector

Since the probability of a devaluation is assumed to be very small, the pre-set price can be considered equal to the price that would maximize profits in the absence of a shock. Assuming that the firm is committed to satisfying any demand at the chosen price, profits in cases of price adjustment Π_a (menu costs not yet subtracted) and without price adjustment Π_n can be computed. The difference between Π_a and Π_n yields the firm's private cost of not adjusting (as it is well known, the social cost is higher because of externalities) during the sole relevant period (by assumption the impact of the shock is transitory). To get a sense of its magnitude, it is useful to take the ratio of this difference to profit $\overline{\Pi}$ in the absence of a shock. This yields:

$$\frac{\Pi_a - \Pi_n}{\overline{\Pi}} = (w_r)^{-\frac{\alpha(\mu-1)}{\mu\left[1-(1-\frac{1}{\mu})\alpha\right]}} Z^{\frac{1}{\mu\left[1-(1-\frac{1}{\mu})\alpha\right]}} - \left[1-\alpha\left(1-\frac{1}{\mu}\right)\right]^{-1}\left[Z - w_r Z^{\frac{1}{\alpha}}\left(1-\frac{1}{\mu}\right)\alpha\right], \tag{1}$$

where $Z = (P_{Nr})^{\mu-\rho}(P_r)^{\rho} C_r$ and $C_r = 1$ (except for section 6, real consumption will be assumed, for simplicity's sake, to be constant).

If G_N is the cumulative of non-tradables firms' menu costs (expressed as a proportion of $\overline{\Pi}$), then the proportion α_N of non-tradables firms that adjust their prices is given by $\alpha_N = G_N\left(\frac{\Pi_a-\Pi_n}{\overline{\Pi}}\right)$. Since G_N is assumed to be degenerated (all firms have the same menu cost), α_N is either 0 or 1 depending on whether $\frac{\Pi_a-\Pi_n}{\overline{\Pi}}$ is smaller or larger than the common menu cost (α_N may take an intermediate value in the special case when $\frac{\Pi_a-\Pi_n}{\overline{\Pi}}$ is exactly equal to the common menu-cost, since some firms may adjust while other firms do not).

3.2 Optimization by households

Similarly, the ratio of the household's utility cost of not adjusting $(U_a - U_n)$ to its utility in the absence of a shock $(\overline{U})$ can be computed, where U_a is the utility in case of wage adjustment (menu costs not yet subtracted) and U_n is the utility if the household does not adjust its wage. This household's private

[14] w_r for wages, P_r for the price level, P_{Nr} for the price of the non-tradable aggregate, $C_r = \frac{N_r}{P_r}$ for the CES index of the consumption of tradable and non-tradable goods, N_r for nominal demand, L_r for labor.

cost of not adjusting its wage during the sole relevant period (expressed as a proportion of its utility in the absence of a shock) is:

$$\frac{U_a - U_n}{\overline{U}} = \left[P_r^{-\eta} L_r w_r^{\eta}\right]^{\frac{1+\theta}{1+\eta\theta}} - \left(1 - \frac{1-\frac{1}{\eta}}{1+\theta}\right)^{-1} \left[P_r^{-1} L_r w_r^{\eta} - \left(L_r w_r^{\eta}\right)^{1+\theta} \frac{1-\frac{1}{\eta}}{1+\theta}\right]. \tag{2}$$

If G_w is the cumulative of households' menu costs (expressed as a proportion of $\overline{U}$), then the proportion α_w of households that adjust their wages is given by $\alpha_w = G_w\left(\frac{U_a - U_n}{\overline{U}}\right)$. Again, the cumulative is assumed to be degenerate (all households have the same menu cost).

3.3 Definitions and market clearing conditions

The aggregate wage level is given by $w = \left[\int_0^1 (w_h)^{1-\eta} dh\right]^{\frac{1}{1-\eta}}$ where w_h is the wage set by household h. Knowing that a proportion α_w of households adjust their wages, and knowing which proportion of adjusting households will change their wages, w_r can be computed:

$$w_r = \left\{\alpha_w \left[P_r (L_r)^{\theta} (w_r)^{\eta\theta}\right]^{-\frac{\eta-1}{1+\eta\theta}} + (1-\alpha_w)\right\}^{-\frac{1}{\eta-1}}. \tag{3}$$

A similar computation can be made for the aggregate price for non-tradables, the aggregate price level and aggregate labor:

$$P_{Nr} = \left\{\alpha_N \left[(w_r)^{\alpha} (P_r)^{\rho(1-\alpha)} (C_r)^{1-\alpha} (P_{Nr})^{(\mu-\rho)(1-\alpha)}\right]^{\overline{\frac{1-\mu}{\mu\left[1-(1-\frac{1}{\mu})\alpha\right]}}} + (1-\alpha_N)\right\}^{\frac{1}{1-\mu}}, \tag{4}$$

$$P_r = \left[\frac{(P_{Tr})^{1-\rho}\,\Omega + (P_{Nr})^{1-\rho}}{\Omega+1}\right]^{\frac{1}{1-\rho}},$$

where $\Omega = \frac{\overline{P}_T \overline{C}_T}{\overline{P}_N \overline{C}_N}$,

$$L_r = \alpha_N \left[(w_r)^{-\mu} (P_{Nr})^{\mu-\rho} (P_r)^{\rho} C_r\right]^{\overline{\frac{1}{\mu\left[1-(1-\frac{1}{\mu})\alpha\right]}}} + (1-\alpha_N)\left[(P_{Nr})^{\mu-\rho} (P_r)^{\rho} C_r\right]^{\frac{1}{\alpha}}. \tag{6}$$

4 Cases when wages are not adjusted

A change in the price of tradables has an impact on the price of non-tradables through the goods market (if $\rho \neq 0$) and through wages (I assume C_r exogenous and equal to 1). If households' menu costs are high enough for wages to stay constant (and thus $w_r = 1$), then there is only the goods market channel left, through which the impact might not be strong if the elasticity of substitution ρ is close enough to 0 (BER set $\rho = 0.4$)[15].

Formally, equation (4) becomes:

$$(P_{Nr})^{1-\mu} = \alpha_N \left\{ \left[\frac{\left(\frac{P_{Tr}}{P_{Nr}}\right)^{1-\rho}\Omega+1}{\Omega+1} \right]^{\frac{\rho(1-\alpha)}{1-\rho}} (P_{Nr})^{\mu(1-\alpha)} \right\}^{\frac{1-\mu}{\mu\left[1-(1-\frac{1}{\mu})\alpha\right]}} + (1-\alpha_N),$$

where α_N is given by equation (1).

- If the menu costs of firms are sufficiently high, then $P_{Nr} = 1$. This happens if their menu costs are higher than their private costs of not adjusting their prices, which is equal (according to §3.1) to the following critical value: $Z^{\frac{1}{\mu\left[1-(1-\frac{1}{\mu})\alpha\right]}} - \left[1-\alpha\left(1-\frac{1}{\mu}\right)\right]^{-1}\left[Z - Z^{\frac{1}{\alpha}}\left(1-\frac{1}{\mu}\right)\alpha\right]$ where $Z = (P_r)^{\rho}$ and $P_r = \left[\frac{(P_{Tr})^{1-\rho}\Omega+1}{\Omega+1}\right]^{\frac{1}{1-\rho}}$. The assumption that wages are not adjusted implies (according to §3.2) that households' menu costs are larger than $\left[(P_r)^{-\eta} L_r\right]^{\frac{1+\theta}{1+\eta\theta}} - \left(1-\frac{1-\frac{1}{\eta}}{1+\theta}\right)^{-1}\left\{(P_r)^{-1} L_r - (L_r)^{1+\theta}\frac{1-\frac{1}{\eta}}{1+\theta}\right\}$ where $L_r = (P_r)^{\frac{\rho}{\alpha}}$ and $P_r = \left[\frac{(P_{Tr})^{1-\rho}\Omega+1}{\Omega+1}\right]^{\frac{1}{1-\rho}}$.

- If non-tradables firms have small enough menu costs then they will change their prices. Then $\alpha_N = 1$ (I assume that firms' menu costs are strictly smaller than the critical value and do not discuss the case in which they are exactly equal to the critical value). Equation (4) becomes: $(P_{Nr})^{\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}(\Omega+1) = \left(\frac{P_{Tr}}{P_{Nr}}\right)^{1-\rho}\Omega + 1$. This implicit equation for P_{Nr} has only one solution. Knowing P_{Nr} and w_r, critical menu costs can be computed.

[15] Intuitively, if $w_r = 1$ and $\rho = 0$, then we should have $P_{Nr} = 1$ since there is no open channel left. Formally, equation (4) becomes in this case: $P_{Nr}^{1-\mu} = \alpha_N P_{Nr}^{(1-\mu)\frac{1-\alpha}{1-\alpha+\frac{\alpha}{\mu}}} + 1 - \alpha_N$ which has $P_{Nr} = 1$ as unique solution.

Numerical example

Let's numerically evaluate P_{Nr} and the critical menu costs for the following calibration:

Table 1: Calibration

Value	Justification
$\rho = 0.4$	BER. They quote Stockman and Tesar (1995), Lorenzo, Aboal and
	Osimani (2003), and Gonzales-Rozada and Neumeyer (2003).
$\theta = 0.25$	BER. This value implies a labor-supply elasticity that coincides with
	the standard value used in the real business-cycle literature.
$\Omega = 1/3$	Implies that the pre-devaluation share of tradable goods in CPI
	(distribution costs not included) is 25%. Burnstein et al. (2005a)
	argue that tradable goods (distribution costs included) account for
	roughly 50% of the CPI basket, but that about half of their costs
	are distribution costs. This leaves a share of 25% for pure
	tradable goods.
$\mu = 6$	This is a benchmark in the literature.
$\eta = 2$	Naknoi (2005) referring to the study by Huang and Liu (2002) who
	find that it can vary from 2 to 4.
$\alpha = 2/3$	Is a realistic value for the share of labor income in GDP.
$C_r = 1$	Real consumption is assumed not to be affected by the devaluation.
$P_{Tr} = 2$	The devaluation shock is such that the domestic currency loses
	half of its value and the price of tradables doubles.

For this calibration, I find the following values. Non-tradables firms do not change their prices if their menu costs are larger than 2.4×10^{-3}. In this case, households' menu costs must be larger than 2.8×10^{-2} for households not to change their wages. If non-tradables firms have menu costs small enough to change their prices, then they adjust their prices by a factor $P_{Nr} = 1.04$. The critical firms' menu cost, below which they adjust their prices, is 4.3×10^{-2}. This critical value is larger than the critical value obtained in the case that other non-tradables firms do not adjust (2.4×10^{-3}) since adjustment of other non-tradables prices create an extra incentive for a given non-tradables firm to adjust its price. This means that for a menu cost between 2.4×10^{-3} and 4.3×10^{-2} a non-tradables firm will adjust its price or not depending on whether other firms do or not (multiple equilibria). The menu cost of households has to be larger than 3.3×10^{-2} for them not to increase their wage although the prices of non-tradables have increased.

For this calibration, this numerical example shows that a low small firms' menu cost is enough to be consistent with non-tradables firms not changing their prices. Since the price of non-tradables does not change, a quite low households' menu cost is enough for households not changing their wage to be an equilibrium. Moreover, I also obtain the possibility that the price of non-tradables will change albeit by a small amount (this result was not obtained in the BER model).

5 Cases when wages are adjusted

Let's assume now that households' menu costs are small enough for all wages to adjust. Then equation (3) yields $w_r = P_r (L_r)^\theta$ where the amount of labor depends, according to (6), on the quantities the firms want to produce, and thus on the prices they set.

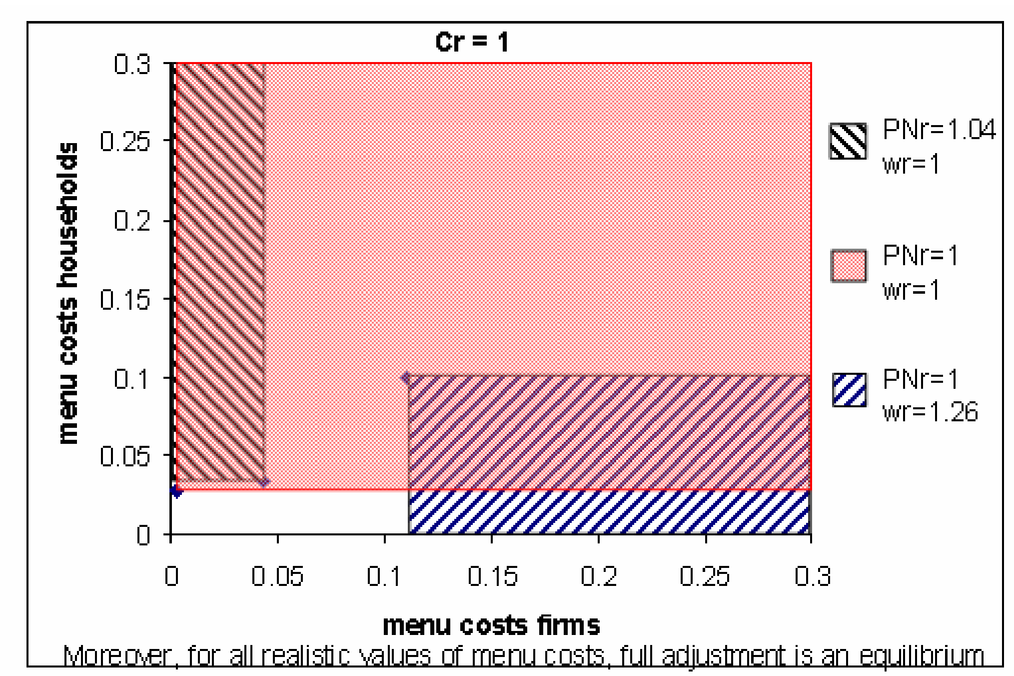
- If non-tradables firms have large enough menu costs to prevent them from adjusting their prices, then $P_{Nr} = 1$, $L_r = (P_r)^{\frac{\rho}{\alpha}}$, $w_r = (P_r)^{1+\frac{\rho}{\alpha}\theta}$ and the critical menu costs under which these results yield can be computed.

- Non-tradables firms will adjust if they have small enough menu costs. In this case equations (3) and (4) yield $(P_{Nr})^{\alpha+\rho(1-\alpha)} = (w_r)^\alpha (P_r)^{\rho(1-\alpha)}$ and $w_r = P_r \left[(w_r)^{-\mu} (P_{Nr})^{\mu-\rho} (P_r)^\rho\right]^{\frac{\theta}{\mu\left[1-(1-\frac{1}{\mu})\alpha\right]}}$. Plugging equation (5) for P_r into these equations yields two curves in the plane $< P_{Nr}; w_r >$. The solution is the intersection. Then knowing P_{Nr} and w_r, the critical menu costs under which these results yield can be computed.

For the above calibration I get the following values. A non-tradables firm will not adjust its price when other non-tradables firms do not adjust if its menu cost is larger than 0.11. This critical value is higher than what it was when households did not adjust their wages since wage adjustment creates an additional incentive for firms to adjust their prices (households multiply their wages by a factor 1.3 which is smaller than the exchange-rate shock but is still large enough to create a big incentive for firms to change their prices). This critical value is so large that in this model it is very unlikely that a firm will not adjust its price while the households are adjusting their wages. This is the case even when the other non-tradables firms do not adjust their prices. If they do, then the incentive to join them is even greater. I find that for any realistic menu-cost values, an agent will always adjust when all the other agents (households and non-tradables firms) do, and in this case price and wage adjustments are complete. Thus, for this calibration I do not get sticky prices for low firms' menu costs if households' menu costs are small enough for them to change their wages. Wage stickiness was crucial to get the results of the previous section.

The equilibria discussed in this and in the preceding section are shown as a function of firms' and households' menu costs in the Figure 1 (as mentioned below, other equilibria exist but they are unstable).

Figure 1: Stable equilibria as a function of menu costs for $C_r = 1$

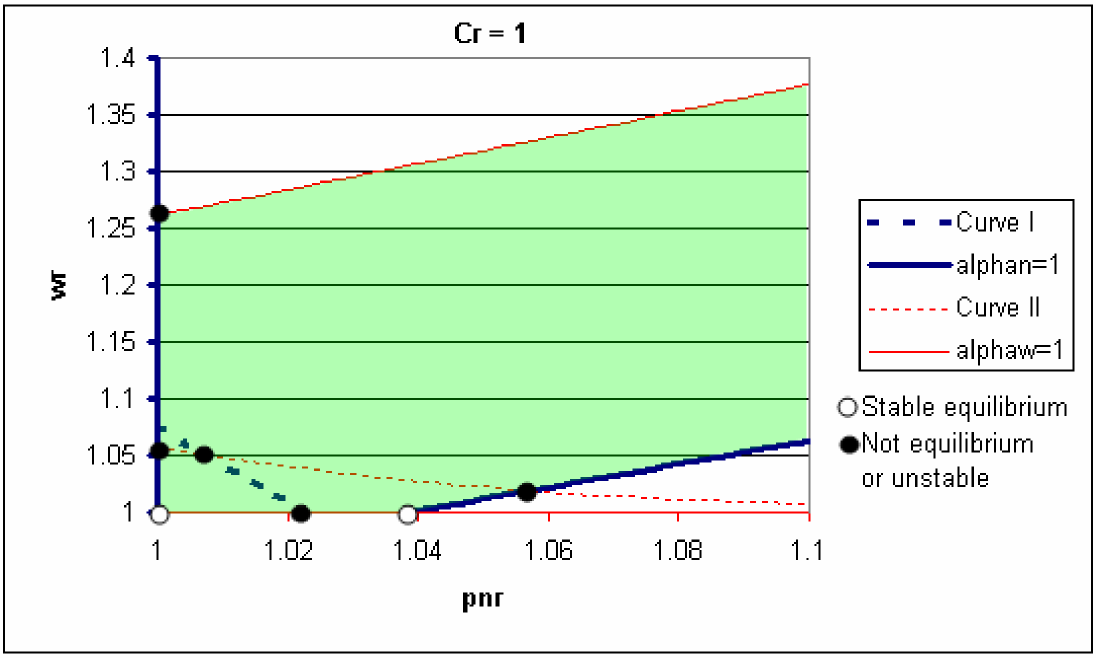


$\rho = 0.4$, $\theta = 0.25$, $\Omega = 1/3$, $\mu = 6$, $\eta = 2$, $\alpha = 2/3$, $C_r = 1$, $P_{Tr} = 2$.

This figure shows that full adjustment is an equilibrium for all realistic values of menu costs. If menu costs are sufficiently small, this is the only equilibrium. However, there are multiple equilibria for some larger menu costs. For these parameter values, the equilibrium (hatched slanting to the right), at which it is possible that the prices of non-tradable goods do not adjust while the wages adjust, exists only for unrealistically high firms' menu costs. But it is possible that neither the prices of non-tradables nor wages adjust (shaded area). It is also possible that wages do not adjust while prices do adjust (hatched slanting to the left).

This figure gives the equilibria as a function of firms' and households' menu costs. A figure showing the equilibria in the plane $< P_{Nr}; w_r >$ can also be drawn. If the menu cost of firms is 2% and the menu cost of households is 4%, then I get Figure 2:

Figure 2: Potential equilibria (zoom) for $C_r = 1$



$\rho = 0.4$, $\theta = 0.25$, $\Omega = 1/3$, $\mu = 6$, $\eta = 2$, $\alpha = 2/3$, $C_r = 1$, $P_{Tr} = 2$, firms' menu costs = 2%, households' menu costs = 4%.

In this figure there are two types of curves: the ones in bold focus on non-tradables firms while the other ones focus on households. There are three curves in bold. One curve corresponds to the case in which all firms adjust ($\alpha_N = 1$). The vertical axes ($P_{Nr} = 1$) correspond to the case in which non-tradables firms do not adjust ($\alpha_N = 0$). Finally, Curve I corresponds to the case in which the menu cost is exactly equal to the private cost of not adjusting (and thus some non-tradables firms might choose to adjust while others choose not to adjust). If a point $< P_{Nr}; w_r >$ is located above this curve, then the cost of not adjusting is larger than the menu cost and all firms would prefer to adjust. At a point located below this curve, no non-tradables firm would prefer to adjust. Similarly, there are three curves not in bold (horizontal axe included). The shaded area is the locus of points for which $0 \leq \alpha_N \leq 1$ and $0 \leq \alpha_w \leq 1$. Thus, points outside the shaded area should be disregarded. Even a point in the shaded area cannot be an equilibrium if it is not at the intersection between a curve in bold and another curve. But the reverse is not true: an intersection is not necessarily an equilibrium. Whether a given intersection is or is not an equilibrium depends on the values of the menu costs for firms and households. For example, the intersection between $P_{Nr} = 1$ and $\alpha_w = 1$ is not an equilibrium because it is located above Curve I (the firms prefer to adjust their prices rather than stay at $P_{Nr} = 1$). But if the menu cost of firms became sufficiently high, then Curve I would move to the upper right-hand corner and would eventually have moved enough for this intersection to be located below that curve. Thus, an intersection is a potential equilibrium in the sense of being an equilibrium for some values of the menu costs. In addition to the intersection shown in Figure 2 (which is a zoom) there is an intersection at $< 2; 2 >$ corresponding to full adjustment.

Figure 2 also helps to discuss the stability of these equilibria. For example the intersection between Curve II and $\alpha_N = 1$ is unstable: starting from a point at the right but still near this intersection on the curve $\alpha_N = 1$, such a point would be above Curve II and all households would want to adjust their wages, increasing wages and prices even more until the equilibrium $< 2; 2 >$ is reached. It can be seen that each of the four equilibria shown in Figure 1 is stable for menu-cost values for which it is indeed an equilibrium.

It may be surprising to see in Figure 1 that no amount of change of one critical value can make up for a change in the other critical value in order to yield the same equilibrium. As an example, let's discuss this for the $P_{Nr} = 1$ & $w_r = 1$ equilibrium. If the households' menu cost is a little bit smaller than the needed critical value then this equilibrium vanishes. No change in the menu cost of firms can make up for it. In Figure 2 it is clear what happens. The point $< P_{Nr}; w_r > = < 1; 1 >$ is located below Curve II when the households' menu cost is equal to 4%. However, if the menu cost of households were small enough, then the point $< P_{Nr}; w_r > = < 1; 1 >$ would be located above Curve II and all households would want to adjust their wages. Increasing the menu cost of firms would move Curve I, but would not change the fact that $< 1; 1 >$ is located above Curve II. Intuitively, if the households' menu cost is too small then households will adjust their wage whatever non-tradables firms do.

One could reply that this example is special: the price of non-tradables should have an impact on whether a household adjusts or not (and by how much), but the price of non-tradables is a given in this example in which non-tradables firms do not adjust. Thus let's consider the equilibrium $< P_{Nr}; w_r > = < 1.04; 1 >$ at which the price of non-tradables adjusts. As before, this equilibrium disappears if the households' menu cost is too small. A decrease of P_{Nr} would indeed decrease the households' cost of not adjusting and could compensate a small decrease in the households' menu cost. But a change in the non-tradables firms' menu cost would have no impact on P_{Nr} except if it was so large that non-tradables firms prefer not to adjust their prices (in which case the equilibrium $< P_{Nr}; w_r > = < 1.04; 1 >$ disappears and the economy would be at $< P_{Nr}; w_r > = < 1; 1 >$). Intuitively, one could expect that the strong impact that a small menu cost change has in this model (when it is near the critical value) is a consequence of the assumption that all firms have the same menu cost and all households as well. I conjecture that if the menu cost distribution is not degenerated, then a change in the firms' menu cost would have an impact on the proportion of firms that adjust (and maybe on the price chosen by adjusting firms) and thus on the aggregate price of non-tradables. In this case there would exist a continuum of equilibria (differing by P_{Nr} and/or w_r) and a little change of the average menu cost would usually not have a strong impact.

6 The importance of central-bank credibility

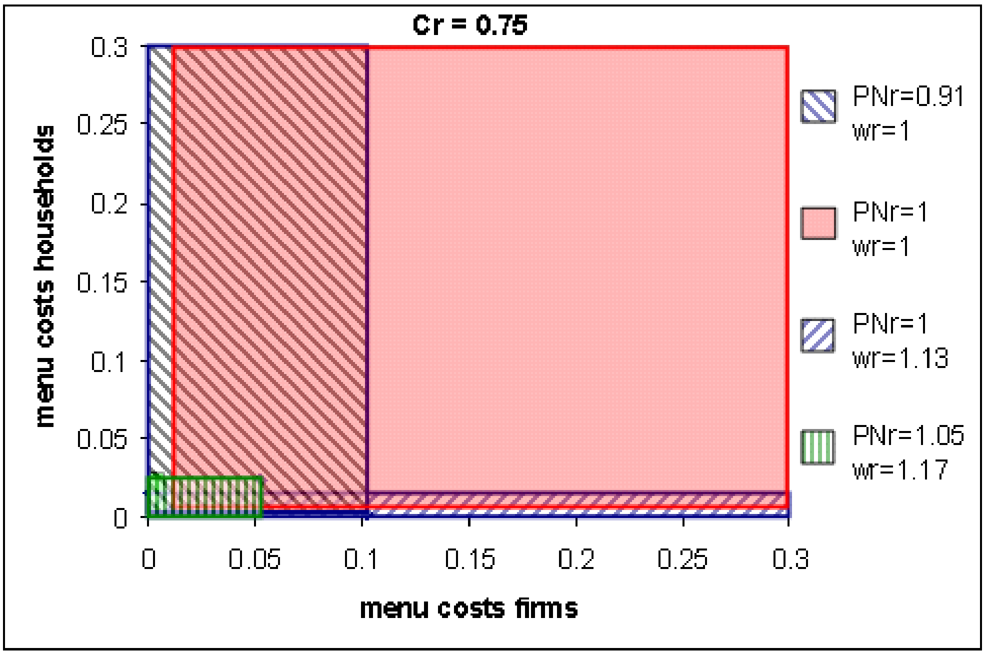
The central bank sets the real consumption of workers through setting money supply ($C_r = M/P$).[16] The central bank chooses C_r such that prices of non-tradables do not increase (or do not increase much) since it wants to avoid the exchange-rate shock leading to inflation. The problem is that there are usually multiple equilibria. The set of equilibria depends on C_r. If the central bank is credible, it only needs its preferred equilibrium to belong to the set of equilibria (it will become the focal equilibrium). This, however, is not sufficient if the central bank is not credible. In that case, even

[16] I assume that monetary policy can indirectly determine real consumption C by choosing the nominal money supply M since $C = \frac{M}{P}$. Notice that all our equilibrium equations are still valid if we consider that M rather than C_r is exogenous. The reason is that the first-order equations are derived assuming that agents take P as exogenous. Thus $\frac{M}{P}$ will be treated by the optimizing agents as exogenous as C_r.

if its preferred equilibrium belongs to the set of equilibria, agents will not necessarily focus on it. Thus, when the central bank is lacking in credibility, it will have to generate a large enough recession (choose C_r small enough) that the equilibrium it wants to avoid no longer belongs to the set of equilibria.

Let's assume that the monetary authorities want to avoid inflation and thus want firms of the non-tradables sector to choose not to adjust their prices. Monetary authorities can achieve this goal by choosing a C_r such that not (or barely) changing prices is the only optimal decision for price-setters. This is always possible by choosing C_r sufficiently small. Figure 3 reproduces Figure 1 for $C_r = 0.75$ instead of 1, and shows how these stable equilibria depend on menu costs.

Figure 3: Stable equilibria for $C_r = 0.75$

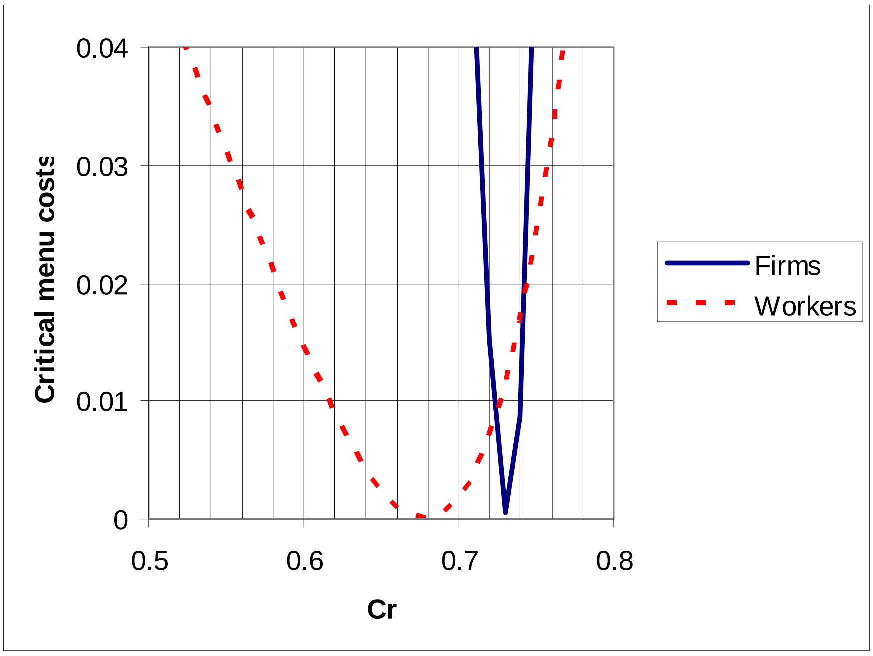


$\rho = 0.4$, $\theta = 0.25$, $\Omega = 1/3$, $\mu = 6$, $\eta = 2$, $\alpha = 2/3$, $C_r = 0.75$, $P_{Tr} = 2$.

Compared with Figure 1, the most important difference is that the area corresponding to the menu cost values for which there is an equilibrium, at which all agents adjust, has shrunk. In Figure 1 it took up the entire area visible on the graph (I didn't designate the corresponding zone in order not to overburden the figure). Here, the corresponding zone (hatched vertically) is smaller. For example, this equilibrium is no longer obtained if the firms' menu cost=2% and the households' menu cost=4%. Moreover, producers of non-tradables do not change their prices much when all agents adjust: they increase them by only 5%. Thus, when $C_r = 0.75$ there is not much danger of an increase of P_{Nr} and this increase would be small in any case (but there is a possibility of a decrease of P_{Nr}: the surface (hatched slanting to the left) corresponding to $P_{Nr} = 0.91$).

If the central bank wants to avoid an increase of P_{Nr}, it can do so by choosing C_r low enough. But how low C_r has to be (that is, how large the recession needs to be) depends on the credibility of the central bank. If the central bank is not credible, then it will need to chose a C_r low enough for no adjustment to be the only possible equilibrium at the given menu cost values (or at least to exclude equilibria which imply an inordinately large increase of P_{Nr}). If the central bank is credible, then it does not need to reduce C_r that much (depending on the menu costs, it might not need to reduce C_r at all) : it is enough that not adjusting belong to the multiple equilibria. This implies that two identical countries that differ only by the credibility of their central bank can end up with different C_r after an identical exchange-rate shock. For example, if the menu cost is 2% for firms and 4% for households, then, with the above parameter values, a credible central bank can keep real consumption constant while a central bank that does not benefit from this credibility would have to decrease real consumption by a large amount. One could ask what the maximum value of C_r would be such that the equilibrium at which all agents adjust is excluded for the above parameter values. Figure 4 (a zoom on the relevant zone) shows, as a function of C_r, the critical menu costs for firms and households such that all agents adjust only if menu costs are smaller than these critical values.

Figure 4: Critical menu costs for the equilibrium at which all agents adjust



$\rho = 0.4$, $\theta = 0.25$, $\Omega = 1/3$, $\mu = 6$, $\eta = 2$, $\alpha = 2/3$, $P_{Tr} = 2$.

To exclude the equilibrium at which all agents adjust, it is enough that one of the menu costs is above the corresponding critical value. When the menu cost of firms is 2% and the menu cost of households is 4%, then the equilibrium at which all agents adjust is excluded at $C_r = 0.76$. Notice that the curves are steep: a small change in C_r can have a large impact on the menu-cost values compatible with all agents adjusting.

7 Conclusion

I have shown that menu costs can explain not only why the price of non-tradables may remain unchanged after a large devaluation, but also why it may change by a small amount. I usually obtain multiple equilibria. If monetary policy is credible, the equilibrium preferred by the central bank will be selected. If monetary policy is not credible, then the central bank will have to generate a recession large enough that the equilibrium it wants to be certain of avoiding is no longer one of the multiple equilibria.

This paper could be extended in several directions. It would be interesting to extend the model to general equilibrium (for example, foreign demand for tradable goods could be modelized and real consumption endogenized), to relax simplifying assumptions (for example, the assumption that prices are fully flexible following the first period after the shock could be dropped), to introduce savings and the interest rate (and maybe a monetary policy using this interest rate as an instrument), to model the cause of the exchange-rate shock (and its possible links to monetary policy), to integrate substantial dynamics, and to allow for different firms having different menu costs (idem for households). One may also want to explain in terms of menu costs faced by producers of tradables why the price of tradables adjusts whereas the price of non-tradables adjusts only to a small extent (intuitively, one could expect that a firm's opportunity cost of not adjusting its price is higher in the tradables sector, in which the exchange-rate shock is felt more directly).

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Appendix: Equilibrium equations

Households

In the absence of shocks, a household maximizes its utility $\overline{U} = \overline{c}_h - \gamma \frac{\overline{l}_h^{1+\theta}}{1+\theta}$ under its budget constraint $\overline{c}_h \overline{P} = \overline{w}_h \overline{l}_h$ (assuming that workers have only labor income), where h stands for household h, the line over a variable indicates that it refers to the case without shocks, and $\overline{l}_h = \left(\frac{\overline{w}_h}{\overline{w}}\right)^{-\eta} \overline{L}$. Thus the utility is $\overline{U} = \frac{\overline{w}\left(\frac{\overline{w}_h}{\overline{w}}\right)^{1-\eta} \overline{L}}{\overline{P}} - \gamma \frac{\left[\left(\frac{\overline{w}_h}{\overline{w}}\right)^{-\eta} \overline{L}\right]^{1+\theta}}{1+\theta}$. The first-order condition (with respect to w_h) is given by: $(\eta - 1) \frac{\overline{w}}{\overline{P}} \left(\frac{\overline{w}_h}{\overline{w}}\right)^{1-\eta} \overline{L} = \eta\gamma \left[\left(\frac{\overline{w}_h}{\overline{w}}\right)^{-\eta} \overline{L}\right]^{1+\theta}$. Thus: $\overline{w}_h = \left(\overline{P} \frac{\eta}{\eta-1} \gamma \overline{L}^{\theta} \overline{w}^{\eta\theta}\right)^{\frac{1}{1+\eta\theta}}$, $l_h = \left(\overline{P} \frac{\eta}{\eta-1} \gamma\right)^{-\frac{\eta}{1+\eta\theta}} \overline{L}^{\frac{1}{1+\eta\theta}} \overline{w}^{\frac{\eta}{1+\eta\theta}}$ and $\overline{c}_h = \overline{P}^{-\eta \frac{1+\theta}{1+\eta\theta}} \left(\frac{\eta}{\eta-1}\gamma\right)^{\frac{1-\eta}{1+\eta\theta}} \overline{L}^{\frac{1+\theta}{1+\eta\theta}} \overline{w}^{\eta \frac{1+\theta}{1+\eta\theta}}$. In the absence of shocks, the utility is given at equilibrium by: $\overline{U} = \frac{\overline{w}\left(\frac{\overline{w}_h}{\overline{w}}\right)^{1-\eta} \overline{L}}{\overline{P}} - \gamma \frac{\left[\left(\frac{\overline{w}_h}{\overline{w}}\right)^{-\eta} \overline{L}\right]^{1+\theta}}{1+\theta} = \frac{\eta}{\eta-1}\gamma \left[\left(\frac{\overline{w}_h}{\overline{w}}\right)^{-\eta} \overline{L}\right]^{1+\theta} - \gamma \frac{\left[\left(\frac{\overline{w}_h}{\overline{w}}\right)^{-\eta} \overline{L}\right]^{1+\theta}}{1+\theta} = \frac{1+\eta\theta}{\eta-1}\gamma \frac{\left[\left(\frac{\overline{w}_h}{\overline{w}}\right)^{-\eta} \overline{L}\right]^{1+\theta}}{1+\theta} = \frac{1+\eta\theta}{\eta-1}\gamma \frac{\left(\overline{P}\frac{\eta}{\eta-1}\gamma \overline{w}^{-1}\right)^{-\eta\frac{1+\theta}{1+\eta\theta}} \overline{L}^{\frac{1+\theta}{1+\eta\theta}}}{1+\theta}$.

If there is a shock, either the household adjusts its wage or it does not adjust. Let $\widetilde{w}_h$ be the wage chosen by households that adjust, and w the aggregate wage after the shock, L the aggregate labor and P the aggregate price. If a household adjusts, it will choose $\widetilde{w}_h = \left(P \frac{\eta}{\eta-1} \gamma L^{\theta} w^{\eta\theta}\right)^{\frac{1}{1+\eta\theta}} = \overline{w}_h \left[\frac{P}{\overline{P}} \left(\frac{L}{\overline{L}}\right)^{\theta} \left(\frac{w}{\overline{w}}\right)^{\eta\theta}\right]^{\frac{1}{1+\eta\theta}} = \overline{w}_h Y^{\frac{\theta}{1+\eta\theta}}$, where $Y \equiv P_r^{\frac{1}{\theta}} L_r w_r^{\eta}$, where all variables with index "r" stand for the ratio of that variable after the shock to its value before the shock. Its utility will be: $U_a = \frac{1+\eta\theta}{\eta-1}\gamma \frac{\left(P\frac{\eta}{\eta-1}\gamma w^{-1}\right)^{-\eta\frac{1+\theta}{1+\eta\theta}} L^{\frac{1+\theta}{1+\eta\theta}}}{1+\theta}$. If the household does not adjust its wage, its utility will be: $U_n = \frac{w\left(\frac{\overline{w}_h}{w}\right)^{1-\eta} L}{P} - \gamma \frac{\left[\left(\frac{\overline{w}_h}{w}\right)^{-\eta} L\right]^{1+\theta}}{1+\theta}$

$$= \frac{w\left(\overline{P}\frac{\eta}{\eta-1}\gamma \overline{L}^{\theta} \frac{\overline{w}^{\eta\theta}}{w^{1+\eta\theta}}\right)^{\frac{1-\eta}{1+\eta\theta}} L}{P} - \gamma \frac{\left(\overline{P}\frac{\eta}{\eta-1}\gamma \overline{L}^{\theta} \frac{\overline{w}^{\eta\theta}}{w^{1+\eta\theta}}\right)^{-\eta\frac{(1+\theta)}{1+\eta\theta}} L^{1+\theta}}{1+\theta}$$

$$= \frac{w\left(\frac{\overline{w}^{\eta\theta}}{w^{1+\eta\theta}}\right)^{\frac{1-\eta}{1+\eta\theta}} L}{P} \left(\overline{P}\frac{\eta}{\eta-1}\gamma \overline{L}^{\theta}\right)^{\frac{1-\eta}{1+\eta\theta}} - \gamma \frac{\left(\overline{P}\frac{\eta}{\eta-1}\gamma \overline{L}^{\theta} \frac{\overline{w}^{\eta\theta}}{w^{1+\eta\theta}}\right)^{-\eta\frac{1+\theta}{1+\eta\theta}} L^{1+\theta}}{1+\theta}.$$

Plugging the above utilities into $\frac{U_a - U_n}{\overline{U}}$ yields for the household's opportunity cost expressed as a proportion of the utility before the shock:

$$\frac{U_a - U_n}{\overline{U}} = \left[P_r^{-\eta} L_r w_r^{\eta}\right]^{\frac{1+\theta}{1+\eta\theta}} - \left(1 - \frac{1-\frac{1}{\eta}}{1+\theta}\right)^{-1} \left[P_r^{-1} L_r w_r^{\eta} - \left(L_r w_r^{\eta}\right)^{1+\theta} \frac{1-\frac{1}{\eta}}{1+\theta}\right]$$
$$= (P_r)^{-\frac{1}{\theta}-1} \left\{ Y^{\frac{1+\theta}{1+\eta\theta}} - \left(1 - \frac{1-\frac{1}{\eta}}{1+\theta}\right)^{-1} \left(Y - Y^{1+\theta} \frac{1-\frac{1}{\eta}}{1+\theta}\right) \right\}.$$

A household that adjusts its wage will work
$\widetilde{l}_{h,a} = \left(\frac{\widetilde{w}_h}{w}\right)^{-\eta} L = \left(\frac{\widetilde{w}_h}{\overline{w}_h} \frac{\overline{w}}{w}\right)^{-\eta} \frac{L}{\overline{L}} \left(\frac{\overline{w}_h}{\overline{w}}\right)^{-\eta} \overline{L} = \left(Y^{\frac{\theta}{1+\eta\theta}}\right)^{-\eta} (w_r)^{\eta} L_r \bar{l}_h = \left[P_r^{-\eta} L_r w_r^{\eta}\right]^{\frac{1}{1+\eta\theta}} \bar{l}_h.$
A household that does not adjust its wage will work
$\widetilde{l}_{h,n} = \left(\frac{\overline{w}_h}{w}\right)^{-\eta} L = \left(\frac{\overline{w}}{w}\right)^{-\eta} \frac{L}{\overline{L}} \left(\frac{\overline{w}_h}{\overline{w}}\right)^{-\eta} \overline{L} = w_r^{\eta} L_r \bar{l}_h.$

Producers of non-tradables

In the absence of shocks, producers of non-tradables maximize their profit $\overline{\Pi} = \overline{P}_{N,i} \bar{c}_{N,i} - \overline{w} \left(\frac{\bar{c}_{N,i}}{A_N}\right)^{\frac{1}{\alpha}}$, where i stands for firm i, and $\bar{c}_{N,i}$ is the production of firm i. Since the non-tradable good market (like all other markets) is assumed to be cleared, $\bar{c}_{N,i}$ is also the quantity of products of firm i that consumers want to consume at price $\overline{P}_{N,i}$, and is equal to $\bar{c}_{N,i} = (1-\nu) \left(\frac{\overline{P}_{N,i}}{\overline{P}_N}\right)^{-\mu} \left(\frac{\overline{P}_N}{\overline{P}}\right)^{-\rho} \overline{C}$.

The first-order condition (with respect to $\overline{P}_{N,i}$) is given by: $(\mu - 1) \overline{P}_{N,i} \bar{c}_{N,i} = \frac{\mu}{\alpha} \overline{w} A_N^{-\frac{1}{\alpha}} \left(\bar{c}_{N,i}\right)^{\frac{1}{\alpha}}$.

Thus $\overline{P}_{N,i} = \left\{ \frac{\mu}{(\mu-1)\alpha} \overline{w} A_N^{-\frac{1}{\alpha}} \left[(1-\nu) \overline{P}^{\rho} \left(\overline{P}_N\right)^{\mu-\rho} \overline{C}\right]^{\frac{1}{\alpha}-1} \right\}^{\frac{1}{1+\mu\left(\frac{1}{\alpha}-1\right)}}$, $\bar{c}_{N,i} = (1-\nu) \left(\frac{\overline{P}_{N,i}}{\overline{P}_N}\right)^{-\mu} \left(\frac{\overline{P}_N}{\overline{P}}\right)^{-\rho} \overline{C}$.

In the absence of shocks the profit of the firm at equilibrium is

$$\overline{\Pi} = \overline{P}_{N,i} \bar{c}_{N,i} - \overline{w} \left(\frac{\bar{c}_{N,i}}{A_N}\right)^{\frac{1}{\alpha}} = \left(\frac{\mu}{\alpha(\mu-1)} - 1\right) \overline{w} \left(\frac{\bar{c}_{N,i}}{A_N}\right)^{\frac{1}{\alpha}}$$
$$= \left(\frac{\mu}{\alpha(\mu-1)} - 1\right) \overline{w} \left[(1-\nu) \overline{P}^{\rho} \left(\overline{P}_N\right)^{\mu-\rho} \overline{C}\right]^{\frac{1}{\alpha}} A_N^{-\frac{1}{\alpha}} \left(\overline{P}_{N,i}\right)^{-\frac{\mu}{\alpha}}$$
$$= \frac{\left(\frac{\mu}{\alpha(\mu-1)} - 1\right) \overline{w} \left[(1-\nu) \overline{P}^{\rho} \left(\overline{P}_N\right)^{\mu-\rho} \overline{C}\right]^{\frac{1}{\alpha}} A_N^{-\frac{1}{\alpha}}}{\left\{ \frac{\mu}{(\mu-1)\alpha} \overline{w} A_N^{-\frac{1}{\alpha}} \left[(1-\nu) \overline{P}^{\rho} \left(\overline{P}_N\right)^{\mu-\rho} \overline{C}\right]^{\frac{1}{\alpha}-1} \right\}^{\frac{\mu}{\alpha} \frac{1}{1+\mu\left(\frac{1}{\alpha}-1\right)}}}.$$

If there is a shock, either the firm adjusts or it does not adjust. Let $\widetilde{P}_{N,i}$ be the price chosen by a firm that adjusts. If a firm adjusts it will choose

$$\widetilde{P}_{N,i} = \left\{ \frac{\mu}{(\mu-1)\alpha} w A_N^{-\frac{1}{\alpha}} \left[(1-\nu) P^{\rho} (P_N)^{\mu-\rho} C\right]^{\frac{1}{\alpha}-1} \right\}^{\frac{1}{1+\mu\left(\frac{1}{\alpha}-1\right)}}$$
$$= (w_r)^{\frac{\alpha}{\mu\left[1-\left(1-\frac{1}{\mu}\right)\alpha\right]}} Z^{\frac{1-\alpha}{\mu\left[1-\left(1-\frac{1}{\mu}\right)\alpha\right]}} \overline{P}_{N,i}, \text{where } Z = (P_{Nr})^{\mu-\rho} (P_r)^{\rho} C_r.$$

Its production will be
$\widetilde{c}_{N,i,a} = (1-\nu)\left(\frac{\widetilde{P}_{N,i}}{P_N}\right)^{-\mu}\left(\frac{P_N}{P}\right)^{-\rho} C$
$= (w_r)^{-\frac{\alpha}{\left[1-\left(1-\frac{1}{\mu}\right)\alpha\right]}} Z^{\frac{\alpha}{\mu\left[1-\left(1-\frac{1}{\mu}\right)\alpha\right]}} \overline{c}_{N,i}.$

Its profit will be $\Pi_a = \widetilde{P}_{N,i}\widetilde{c}_{N,i} - w\left(\frac{\widetilde{c}_{N,i,a}}{A_N}\right)^{\frac{1}{\alpha}} = \left(\frac{\mu}{\alpha(\mu-1)}-1\right) w \left(\frac{\widetilde{c}_{N,i,a}}{A_N}\right)^{\frac{1}{\alpha}}$
$= \left(\frac{\mu}{\alpha(\mu-1)}-1\right) w \left(\frac{\widetilde{c}_{N,i,a}}{A_N}\right)^{\frac{1}{\alpha}} = \frac{w}{\overline{w}}\left(\frac{\widetilde{c}_{N,i,a}}{\overline{c}_{N,i}}\right)^{\frac{1}{\alpha}}\overline{\Pi}$
$= (w_r)^{-\frac{(\mu-1)\alpha}{\mu\left[1-\left(1-\frac{1}{\mu}\right)\alpha\right]}} Z^{\frac{1}{\mu\left[1-\left(1-\frac{1}{\mu}\right)\alpha\right]}}\overline{\Pi}.$

If the firm does not adjust, it will produce
$\widetilde{c}_{N,i,n} = (1-\nu)\left(\frac{\overline{P}_{N,i}}{P_N}\right)^{-\mu}\left(\frac{P_N}{P}\right)^{-\rho} C = Z\overline{c}_{N,i}.$

Its profit will be $\Pi_n = \overline{P}_{N,i}\widetilde{c}_{N,i,n} - w\left(\frac{\widetilde{c}_{N,i,a}}{A_N}\right)^{\frac{1}{\alpha}}$

$$= \frac{\overline{P}_{N,i}\widetilde{c}_{N,i,n} - w\left(\frac{\widetilde{c}_{N,i,a}}{A_N}\right)^{\frac{1}{\alpha}}}{\left(\frac{\mu}{\alpha(\mu-1)}-1\right)\overline{w}\left(\frac{\overline{c}_{N,i}}{A_N}\right)^{\frac{1}{\alpha}}}\overline{\Pi} = \left[\frac{\overline{P}_{N,i}\widetilde{c}_{N,i,n}}{\left(\frac{\mu}{\alpha(\mu-1)}-1\right)\overline{w}\left(\frac{\overline{c}_{N,i}}{A_N}\right)^{\frac{1}{\alpha}}} - \frac{w\left(\frac{\widetilde{c}_{N,i,a}}{A_N}\right)^{\frac{1}{\alpha}}}{\left(\frac{\mu}{\alpha(\mu-1)}-1\right)\overline{w}\left(\frac{\overline{c}_{N,i}}{A_N}\right)^{\frac{1}{\alpha}}}\right]\overline{\Pi}$$
$$= \left[Z\frac{\overline{P}_{N,i}\overline{c}_{N,i}}{\left(\frac{\mu}{\alpha(\mu-1)}-1\right)\overline{w}\left(\frac{\overline{c}_{N,i}}{A_N}\right)^{\frac{1}{\alpha}}} - \frac{w_r\left(\frac{\widetilde{c}_{N,i,a}}{\overline{c}_{N,i}}\right)^{\frac{1}{\alpha}}}{\left(\frac{\mu}{\alpha(\mu-1)}-1\right)}\right]\overline{\Pi}$$
$$= \left[Z\frac{\frac{1}{\alpha}\frac{\mu}{\mu-1}\overline{w}\left(\frac{\overline{c}_{N,i}}{A_N}\right)^{\frac{1}{\alpha}}}{\left(\frac{\mu}{\alpha(\mu-1)}-1\right)\overline{w}\left(\frac{\overline{c}_{N,i}}{A_N}\right)^{\frac{1}{\alpha}}} - \frac{w_r\left(\frac{\widetilde{c}_{N,i,a}}{\overline{c}_{N,i}}\right)^{\frac{1}{\alpha}}}{\left(\frac{\mu}{\alpha(\mu-1)}-1\right)}\right]\overline{\Pi}$$
$$= \left[1-\left(1-\frac{1}{\mu}\right)\alpha\right]^{-1}\left[Z - w_r\left(\frac{\widetilde{c}_{N,i,a}}{\overline{c}_{N,i}}\right)^{\frac{1}{\alpha}}\left(1-\frac{1}{\mu}\right)\alpha\right]\overline{\Pi}$$
$$= \left[1-\left(1-\frac{1}{\mu}\right)\alpha\right]^{-1}\left[Z - w_r Z^{\frac{1}{\alpha}}\left(1-\frac{1}{\mu}\right)\alpha\right]\overline{\Pi}.$$

Plugging the above profits into $\frac{\Pi_a-\Pi_n}{\overline{\Pi}}$ yields the firm's opportunity cost expressed as a proportion of the profit before the shock:

$$\frac{\Pi_a-\Pi_n}{\overline{\Pi}} = (w_r)^{-\frac{(\mu-1)\alpha}{\mu\left[1-\left(1-\frac{1}{\mu}\right)\alpha\right]}} Z^{\frac{1}{\mu\left[1-\left(1-\frac{1}{\mu}\right)\alpha\right]}} - \left[1-\left(1-\frac{1}{\mu}\right)\alpha\right]^{-1}\left[Z - w_r Z^{\frac{1}{\alpha}}\left(1-\frac{1}{\mu}\right)\alpha\right]$$

$$= Z\left\{\left[(w_r)^{\alpha} Z^{1-\alpha}\right]^{-\frac{(\mu-1)}{\mu\left[1-(1-\frac{1}{\mu})\alpha\right]}} - \left[1-\alpha\left(1-\frac{1}{\mu}\right)\right]^{-1}\left[1-\alpha\left(1-\frac{1}{\mu}\right)\left[(w_r)^{\alpha} Z^{1-\alpha}\right]^{\frac{1}{\alpha}}\right]\right\}.$$

Aggregate variables

- $\mathbf{P}_{Nr}$

$$P_{Nr} = \frac{\left[\alpha_N\left(\widetilde{P}_{N,i}\right)^{1-\mu}+(1-\alpha_N)\left(\overline{P}_{N,i}\right)^{1-\mu}\right]^{\frac{1}{1-\mu}}}{\overline{P}_{N,i}} = \left[\alpha_N\left(\frac{\widetilde{P}_{N,i}}{\overline{P}_{N,i}}\right)^{1-\mu}+(1-\alpha_N)\right]^{\frac{1}{1-\mu}}$$

$$= \left\{\alpha_N\left[(w_r)^{\alpha}(P_r)^{\rho(1-\alpha)}(C_r)^{1-\alpha}(P_{Nr})^{(\mu-\rho)(1-\alpha)}\right]^{\frac{1-\mu}{\mu\left[1-\left(1-\frac{1}{\mu}\right)\alpha\right]}}+(1-\alpha_N)\right\}^{\frac{1}{1-\mu}}$$

- $\mathbf{P}_r$

As usual, $P = \frac{P_N C_N + P_T C_T}{C}$ can be shown to be equal to $\left[\nu P_T^{1-\rho}+(1-\nu)P_N^{1-\rho}\right]^{\frac{1}{1-\rho}}$.

It can also be shown that $\frac{C_T}{C_N} = \frac{\nu}{1-\nu}\left(\frac{P_T}{P_N}\right)^{-\rho}$.

Thus, $P_r = \frac{\left[\nu P_T^{1-\rho}+(1-\nu)P_N^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[\nu \overline{P}_T^{1-\rho}+(1-\nu)\overline{P}_N^{1-\rho}\right]^{\frac{1}{1-\rho}}} = \left[\frac{\frac{\nu}{1-\nu}\left(\frac{\overline{P}_T}{\overline{P}_N}\right)^{-\rho}\frac{\overline{P}_T}{\overline{P}_N}\left(\frac{P_T}{\overline{P}_T}\right)^{1-\rho}+\left(\frac{P_N}{\overline{P}_N}\right)^{1-\rho}}{\frac{\nu}{1-\nu}\left(\frac{\overline{P}_T}{\overline{P}_N}\right)^{-\rho}\frac{\overline{P}_T}{\overline{P}_N}+1}\right]^{\frac{1}{1-\rho}}$

$$= \left[\frac{\frac{\overline{C}_T\overline{P}_T}{\overline{C}_N\overline{P}_N}\left(\frac{P_T}{\overline{P}_T}\right)^{1-\rho}+\left(\frac{P_N}{\overline{P}_N}\right)^{1-\rho}}{\frac{\overline{C}_T\overline{P}_T}{\overline{C}_N\overline{P}_N}+1}\right]^{\frac{1}{1-\rho}} = \left[\frac{\Omega(P_{Tr})^{1-\rho}+(P_{Nr})^{1-\rho}}{\Omega+1}\right]^{\frac{1}{1-\rho}},$$

where $\Omega \equiv \frac{\overline{P}_T\overline{C}_T}{\overline{P}_N\overline{C}_N}$.

- $\mathbf{L}_r$

$$L = \int_0^1 L_i di.$$

Let $L_{i,a} \equiv$ labor used by firms that adjust and $L_{i,n} \equiv$ labor used by firms that do not adjust.

Thus $L_r = \frac{\alpha_N * L_{i,a}+(1-\alpha_N)*L_{i,n}}{\overline{L}}$

$$= \frac{\alpha_N\left(\frac{\widetilde{c}_{N,i,a}}{A_N}\right)^{\frac{1}{\alpha}}+(1-\alpha_N)\left(\frac{\widetilde{c}_{N,i,n}}{A_N}\right)^{\frac{1}{\alpha}}}{\left(\frac{\overline{c}_{N,i}}{A_N}\right)^{\frac{1}{\alpha}}} = \alpha_N\left(\frac{\widetilde{c}_{N,i,a}}{\overline{c}_{N,i}}\right)^{\frac{1}{\alpha}}+(1-\alpha_N)\left(\frac{\widetilde{c}_{N,i,n}}{\overline{c}_{N,i}}\right)^{\frac{1}{\alpha}}$$

$$= \alpha_N\left[(w_r)^{-\frac{\alpha}{\left[1-\left(1-\frac{1}{\mu}\right)\alpha\right]}} Z^{\frac{\alpha}{\mu\left[1-\left(1-\frac{1}{\mu}\right)\alpha\right]}}\right]^{\frac{1}{\alpha}}+(1-\alpha_N)(Z)^{\frac{1}{\alpha}}$$

$$= \alpha_N\left[(w_r)^{-\mu}(P_{Nr})^{\mu-\rho}(P_r)^{\rho}C_r\right]^{\frac{1}{\mu\left[1-\left(1-\frac{1}{\mu}\right)\alpha\right]}}+(1-\alpha_N)\left[(P_{Nr})^{\mu-\rho}(P_r)^{\rho}C_r\right]^{\frac{1}{\alpha}}.$$

- $\mathbf{w}_r$

$$w = \left[\int_0^1 (w_h)^{1-\eta} \, dh \right]^{\frac{1}{1-\eta}}.$$

Thus $w_r = \frac{\left[\alpha_w (\widetilde{w}_h)^{1-\eta} + (1-\alpha_w)(\overline{w}_h)^{1-\eta}\right]^{\frac{1}{1-\eta}}}{\overline{w}_h} = \left[\alpha_w \left(\frac{\widetilde{w}_h}{\overline{w}_h}\right)^{1-\eta} + (1-\alpha_w)\right]^{\frac{1}{1-\eta}}$

$$= \left\{\alpha_w \left[P_r (L_r)^{\theta} (w_r)^{\eta\theta}\right]^{-\frac{\eta-1}{1+\eta\theta}} + (1-\alpha_w)\right\}^{-\frac{1}{\eta-1}}.$$

Part III

Can a Hybrid Sticky-Price and Sticky-Information Model Reconcile Stylized Facts on the Frequency of Individual Price Changes and on Inflation Dynamics?*

Abstract

This paper presents a rational expectations model compatible with the two following stylized facts: i) individual firms' prices change every six months to a year, ii) inflation response to monetary shocks is hump-shaped. The model considered is a hybrid of the sticky-price and sticky-information models of price adjustment. The sticky-price component delivers the first stylized fact directly. I show that the second stylized fact is also satisfied. Under the assumption that firms' price-setting decisions are strategically neutral, the inflation response to a transitory shock to the money-supply growth rate is hump-shaped for the hybrid model, whereas it is monotonic for both the sticky-price and sticky-information models. If the shock is permanent, then this response is hump-shaped for the sticky-information and the hybrid models, whereas it is flat for the sticky-price model.

Keywords: hump-shaped impulse response, inflation persistence, Phillips curve, strategic complementarity.

JEL Classification Number: E31

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1 Introduction

Most macroeconomic models in the economic literature are unable to reconcile the two following stylized facts, at least not without abandoning rational expectations:

- **Frequency of individual price changes**

  The following quotation by Klenow and Willis (2006b) provides a good summary of the literature on the frequency of price changes: "The recent micro empirical literature [...] finds that nominal prices typically change at least once per year. Bils and Klenow (2004) and Klenow and Kryvtsov (2005) report that U.S. consumer prices change every six months or so, on average. Dyne et al. (2005), surveying a spate of recent studies, conclude that Euro Area prices typically change around once per year. Similarly, Taylor (1999) summarized the earlier evidence as saying prices change once a year on average."

- **Inflation dynamics**

  After a monetary shock, it takes more than one year for prices to completely adjust. The impact of a monetary shock on inflation is not only persistent, it is also hump-shaped. Mankiw (2001) argues that there is a broad consensus that shocks to monetary policy have a delayed and gradual effect on inflation. He refers to the traditional emphasis on the "long and variable lags" of monetary policy and the refrain of central bankers that they need to be forward-looking and respond to inflationary pressures even before inflation arises. He also indicates that it shows up in most empirical work. He refers to specific episodes (Paul Volcker started his historic disinflationary policy in the United States in October 1979, but the big declines in inflation came in 1981 and 1982) and to results from standard vector autoregressions.[1] The large VAR literature on the subject also confirms this finding. For example, Christiano and al. (2005) find that "inflation responds in a hump-shaped fashion, peaking after about two years."

It may seem difficult to reconcile both stylized facts within the same model: when prices are kept constant less than one year (as the microeconomic stylized fact requires), it may seem difficult to account for the

[1] He mentions however that there is some debate about when the maximum impact occurs: Bernanke and Mark Gertler (1995) confirm the conventional wisdom that it occurs after a long lag, finding that monetary shocks have no effect on the price level at all during the twelve months after the shock, whereas Rotemberg and Woodford (1997) find shorter lags, with monetary shocks having a large impact after two quarters.

macroeconomic stylized fact that the impact of a monetary shock on inflation persists for more than one year. Taylor (1980) shows, however, that there is endogenous persistence: even if firms change their prices every year, price adjustment will not be complete after one year if price setting is staggered and if there is strategic complementarity in price setting (that is, firms tend to avoid large changes of their prices relative to those of competitors). Chari et al. (2000) respond that staggered price-setting cannot solve the persistence problem. The feature key to their findings is that their dynamic stochastic general equilibrium (DSGE) model yields strategic substitutability rather than strategic complementarity. Woodford (2003) argues that the parameterization of Chari et al. (2000) implies a "considerable degree of strategic substitutability," whereas they would find substantial strategic complementarity if they had taken into account the existence of firm-specific production factors.

Even though assuming a high degree of strategic complementarity may yield a sufficient degree of inflation persistence, Mankiw (2001) notes that a standard sticky-price model is unable to reproduce the hump-shaped response of inflation following a monetary shock. This is why Mankiw and Reis (2002), henceforth MR, have proposed an alternative to the standard sticky-price model: a sticky-information model.[2] The main new feature of their model is that nominal rigidity is not due to the cost of changing price tags and menus, but to the cost of acquiring information in order to re-optimize prices.[3] While the standard sticky-price model features a Calvo staggered price-setting process, motivated by menu costs, in which all firms face the same constant probability of having the opportunity to change prices, MR assume that in each period all firms face the same constant probability of being able to re-optimize current and future prices (henceforth, MR staggered information-updating process). Between two re-optimizations, a firm follows its price plan rather than keeping its price constant (since there is no menu cost, there is no reason to keep prices constant).[4]

[2] Another advantage of the sticky-information model is that in this model, unlike in the standard sticky-price model, anticipated disinflationary policies have no expansionary effects.

[3] As discussed in Ball et al. (2005), imperfect information is a short-cut to the harder task of modeling imperfect information-processing.

[4] In the sticky-information model, firms set their prices at a constant markup over marginal cost. Thus, they do not need to know aggregate variables but only their own marginal costs. This problem might be solved by assuming that firms do not know their own marginal costs. This is an odd assumption that my hybrid model will inherit through its sticky-information component.

Altig et al. (2005) respond to Chari et al. (2000) by taking into account the existence of firm-specific production factors. This generates enough strategic complementarity to yield realistic dynamics of inflation even though firms re-optimize prices on average only every 1.5 quarters. They respond to the argument of Mankiw (2001) by generating the correct inflation impulse response thanks to a deviation from the standard Calvo staggering price-setting process: they assume that firms that cannot re-optimize index their prices to past inflation rather than keeping them constant.[5] Thus, they implicitly assume that the underlying nominal rigidity is not a menu cost but rather imperfect information on nominal variables or imperfect information-processing.

The models of Mankiw and Reis (2002) and Altig et al. (2005) both have their shortcomings. In particular, these two models yield a hump-shaped inflation response at the cost of assuming that prices change every period[6] (they consider a period to be one quarter, as is usually done in this literature), which does not match the microeconomic stylized fact mentioned above. Since this shortcoming stems from the assumption that there are no menu costs, it is natural to try to fix this problem by introducing some menu costs into the model.[7] Mash (2005) combines the standard sticky-price model with a model of the same family as that of Altig et al. (the model of Christiano et al., 2005, which features indexation but not firm-specific capital). Calibrating the weight of both models to match the microeconomic evidence, he finds that the ability of his hybrid model to match the macroeconomic evidence on inflation persistence is severely compromised. But even if it had matched macroeconomic evidence, the deviation from rational expectations would still be problematic. Mash (2005) argues that if firms could choose their degree of indexation optimally, they would choose a value that corresponds to their belief about the actual persistence of inflation, which would lower persistence over time, converging to a stable long-run value of zero for both actual and perceived persistence.

Collard and Dellas (2006) build a rational expectations model compatible with the two stylized facts mentioned above. They assume a standard Calvo

[5] See Minford and Peel (2004), Mash (2005) and Collard and Dellas (2006) for a discussion of the role of this assumption.

[6] Although there is stickiness, prices change every period except in the special, and in the long run unrealistic, case of zero inflation.

[7] See also Collard and Dellas (2006) "In our view, this [assuming Calvo process without indexation] is the more realistic scenario as the evidence on price setting suggests that firms set their prices infrequently and discretely, and in between price jumps, prices remain constant".

price-staggering process completed with imperfect information. They assume that agents learn about the true aggregate state of the economy gradually, using a Kalman filter based on a set of signals on aggregate variables. They find that short-lived misperceptions of the state of the economy limit initial responses while propagating the shocks over time through the real rigidities.

In this paper I propose another alternative. I build a hybrid model incorporating both Calvo staggered price-setting and MR staggered information-updating, thus combining both underlying sources of nominal rigidity, i.e. menu costs and information costs.[8] Such a hybrid model can deliver the above-mentioned microeconomic stylized fact (and also yields heterogeneity of inflation expectations[9]). It avoids having the shortcoming of the sticky-information model, since in the hybrid model prices do not change every period.[10] It also delivers the same average duration between a shock and a firm's first response to it as in the sticky-price model, without having to assume that prices are, on average, kept constant as long as in that model. The reason is that, in the hybrid model, to respond to a shock, firms must not only have an opportunity to change their prices but also need to be informed about the shock. For simplicity and for comparability, I stay as close as possible to MR's framework, although this has some drawbacks, such as my model inheriting the partial-equilibrium feature of MR's model.

[8] There are other papers that combine frictions based on menu costs and information costs. As in Collard and Dellas (2006), one of these frictions is, however, usually neither MR's sticky information nor sticky prices. Rotemberg and Woodford (1997) assume a one-period decision lag in a standard sticky-price model. Woodford (2003) extends this setting to an arbitrary number of lags. Kiley (1996) proposes a hybrid of a sticky-price model (with endogenized probability of price adjustments) and an imperfect-information model, which, however, is different from the sticky-information model. What is usually called the hybrid model in the literature is a model such as that of Gali and Gertler (1999), in which some agents have backward-looking inflation expectations and the rest have rational expectations. Ball (2000) assumes that in forecasting inflation, agents use only an optimal univariate forecasting rule.

[9] Mankiw and al. (2004) argue that the sticky-information model is capable of explaining many features of the observed evolution of both the central tendency and the dispersion of inflation expectations over the past fifty years.

[10] If the money-supply growth rate is zero, then, even in the sticky-information model firms do not change their price every quarter. In this case, the hybrid model would not match the frequency of individual price changes better than the sticky-information model. The case of zero money-supply growth rate is, however, not empirically relevant, since the average money-supply growth rate is usually different from zero. As is often done in this literature (see for example Woodford, 2003), I will use equations linearized around a zero-inflation steady state to discuss cases in which the average long-run inflation is near zero but not necessarily equal to zero.

One challenge for new Keynesian models is to explain data while not assuming too much friction. Thus, adding two kinds of frictions may seem to be counterproductive. However, considering two types of frictions does not necessarily imply a larger overall amount of friction, but may only change the structure of the frictions involved. Moreover, I argue that the sticky information friction is not enough. Something else is necessary (except when inflation is zero) to replicate the empirical fact that prices do not change every period. Adding the Calvo process to MR's model makes it possible to replicate that fact, without necessarily compromising the ability of the model to match the macroeconomic evidence on the inflation response to monetary shocks (depending on the type of shock, it may even improve it).

One could think that a hybrid of the sticky-price and the sticky-information models would yield an average of the macroeconomic performance of the two pure models. If one believes as MR do that the sticky-information model yields better results than the sticky-price model, this would lead to the conclusion that the hybrid model may not fare as well as MR's model in the macroeconomic dimension. I show, however, that the hybrid model is not an average of the two pure models. In the case of strategic neutrality in price-setting,[11] the impulse response of inflation to an unexpected shock to the level of money supply is the same in the sticky-information model as in the sticky-price model.[12] Moreover, their common inflation response is strictly decreasing rather than hump-shaped. However, I will show for this case that the hybrid model is able to generate a hump-shaped inflation response, while both pure models deliver the same strictly decreasing inflation response.

The intuition is the following. In the sticky-price model, firms set their prices (when they have an opportunity to do so) equal to a weighted average of future desired prices (a desired price is the price a firm would choose if it faced no nominal rigidity). In the sticky-information model, when firms get new information, they set a price plan in which the price at each date is equal to the expected desired price. In case of strategic neutrality, however, the desired price does not depend on the aggregate price level but only on the money supply.[13] Now consider an unexpected and once-and-for-all change

[11] That is, a firm's desired price does not depend on the prices set by its competitors. This is an assumption located between those of Woodford (2003) and Chari et al. (2000).

[12] MR, as well as Keen (2005), notice that the ability of a sticky-information model to produce a long delay in the peak inflation response depends critically on the degree of strategic complementarity.

[13] MR note that in the case of strategic neutrality the desired price moves only with the money supply: firms adjust their prices immediately upon learning of the change in policy; as a result, inflation responds quickly (much as it does in the sticky-price model).

in the money-supply level. In this case, all firms set their prices equal to the new long-term equilibrium level as soon as they have the opportunity to change their prices knowing that the shock has taken place (from now on, I refer to this price adjustment as the "first informed price-adjustment"). They will not need to reset them later on. All firms change their prices by the same amount, determined by the difference between the new money-supply level and the old one. Thus inflation at a given date is proportional to the number of firms that have the opportunity to change their prices and have received the information that the shock has occurred. In both the sticky-price model and the sticky-information model, this number decreases since it is a constant fraction of a decreasing set of firms: in the sticky-price model this is the set of the firms that have not yet had the opportunity to change their prices since the shock, whereas in the sticky-information model it is the set of firms that are not yet informed. In the hybrid case, firms setting their first informed prices today are either firms already informed in the last period and receiving the opportunity to change prices today, or firms that were not yet informed in the last period but are receiving information today as well as the opportunity to change prices. The key point is that the number of firms already informed in the last period and receiving the opportunity to change prices today is a hump-shaped function of time. Immediately after the shock, this set is small because only very few firms are informed. After a sufficiently long time, almost all firms are informed, but they have also almost all had the opportunity to change their prices, so the set is small again. In between, this set reaches a maximum.

This example shows that the hybrid model is not simply a weighted average of the two pure models, but can be superior to both. I am not arguing, however, that the sticky-information model cannot deliver a hump-shaped inflation response. Assuming strategic complementarity, MR have found that the sticky-information model delivers a hump-shaped response.[14] I find that this is true even with strategic neutrality if a permanent shock occurs to the money-supply growth rate rather than to the money-supply level. In this

[14] The following intuition explains why in the case of strategic complementarity the responses differ in the sticky-price and the sticky-information models. In the sticky-price model, the inflation response is maximal when the shock occurs, since the incentive to change prices is greatest at this time (later, the economy will be closer to its new equilibrium). In the sticky-information model firms do not need to overshoot their price changes in order to avoid being stuck in the future with prices that are out of line with those of competitors since they set a price plan rather than a price level (they can plan to increase their prices later when more firms are informed of the shock). The inflation response increases until a peak is reached, after which it converges toward zero as more and more firms are informed and most of the adjustment has already taken place.

case, the inflation responses are qualitatively similar in the hybrid and in the sticky-information models, and both are clearly different from the sticky-price model response. Thus, assuming strategic neutrality in price setting, the hybrid and sticky-information models yield, qualitatively, the same inflation impulse response to a permanent shock to the money-supply growth rate. When this shock is more transitory in nature, then the inflation impulse response of the sticky-information model tends to lose its hump, whereas it stays hump-shaped in the hybrid model: if the shock to the growth rate is completely transitory, then it is equivalent to a shock to the level, and the hybrid model delivers a hump-shaped inflation response whereas the two pure models deliver the same strictly decreasing response.

In this paper I assume strategic neutrality in price setting for three reasons. First, there is currently some debate about what degree of strategic complementarity is realistic. As mentioned above, the strategic-neutrality assumption is compatible with the literature. Second, shutting down the strategic-complementarity channel makes it possible to show that a hump-shaped inflation response can be generated even in the absence of strategic complementarity. The strategic-neutrality assumption also makes it easier to show in which sense the hybrid model is different from an average of the two pure models. The third reason is that strategic neutrality is the only case (except in the extreme cases in which the hybrid model reduces to one of the two pure models) in which the hybrid model yields an exact closed-form solution. As a first pass, it therefore seems reasonable to assume strategic neutrality since it is compatible with the literature, yields interesting results and is easier to compute. This however has a cost. Ball and Romer (1990) have shown that nominal frictions alone are not enough to cause business fluctuations generating large welfare losses. This suggests that assuming strategic neutrality would imply large menu costs or small welfare loss. These issues are difficult to discuss in my model since, as in MR, neither menu and information costs nor the utility function are explicit. Another issue is that strategic complementarity is likely to be necessary to get endogenous persistence. Thus, concerning the macro stylized facts, I will focus on the hump-shaped path of the inflation impulse response. I leave for further research the task of explaining endogenous persistence in a hybrid model with α smaller than 1.

On the way to computing inflation impulse responses, I derive the Phillips curve for the hybrid model without, at this stage, assuming strategic neutrality. One novel feature of this Phillips curve is that it involves a new kind of expectation operator. Since all firms are not perfectly informed, it is not surprising that the expectation $\overline{E}_t(\pi_{t+1})$ of next period inflation which enters the Phillips curve is not the expectation $E_t(\pi_{t+1})$ based on the best knowledge available at time t. However, $\overline{E}_t(\pi_{t+1})$ is not the average of aggregate inflation expectations but the average of the firms' expectations about their own price increases.

Dupor et al. (2006) and Klenow and Willis (2006) discuss hybrid models similar to the one in this paper (they propose, however, general equilibrium models). Dupor et al. (2006) find that both sticky prices and sticky information play an important role for inflation dynamics. Their work is more empirically oriented than mine, whereas I stay closer to MR's framework and find closed-form solutions for the impulse response of inflation. Klenow and Willis (2006) focus on making the link between their hybrid model and new microeconomic evidence. They find "modest support" for the sticky-information model and the hybrid model. Paustian and Pytlarczyky (2006) also develop a model merging sticky prices and sticky information, but in their model no firm faces both frictions: some firms face sticky prices, while others face sticky information. This setup allows them to assess the importance of sticky prices versus sticky information in a nested model that reduces to either specification in the extreme cases. They find that the data favors the sticky-price model over the sticky-information model. Besides these three papers, there are several empirical papers comparing the sticky-price and the sticky-information models without actually building a theoretical hybrid model. Their aim is usually to choose the best among the two or more models.[15]

The plan of this paper is as follows. Section 2 discusses the assumptions about when firms receive information or an opportunity to change their prices. Probability distributions resulting from merging the Calvo staggered price-setting process and the MR staggered information-updating process are computed (for example, the distribution of the time of the first informed price-adjustment after a shock). This time-dependent process must be imbedded in an economic environment in order to yield the magnitude of price changes (rather than their timing only). Section 3 presents the basic equations of this environment, staying as close as possible to MR's framework. Section 4 derives the Phillips curve. Section 5 focuses on the case of strategic neutrality in price setting and presents the inflation impulse response in the cases of three unanticipated shocks: a transitory shock to the money-supply growth rate (or equivalently a permanent shock to the money-supply level),

[15] For example: Keen (2005), Korenok (2005), Korenok and Svanson (2006), Laforte (2005) and Trabandt (2006).

a permanent shock to the money-supply growth rate, and the intermediate case of a persistent but not permanent shock to the money-supply growth rate. Section 6 presents concluding remarks.

2 The hybrid price-setting and information-updating process

This section merges Calvo staggered price-setting and MR staggered information-updating and computes two relevant distributions. The way I merge these two processes is very simple, perhaps the most obvious way to model a firm facing both kinds of nominal rigidities (menu costs and imperfect information).

The Calvo sticky-price process assumes that each firm is always perfectly informed but each period it faces a constant probability λ of being exogenously given an opportunity to change its price (prices are kept constant between two such opportunities). One possible implicit story behind this assumption is that firms are hit by random idiosyncratic shocks, and because of high menu costs, they change their prices only when such a shock happens (assuming that idiosyncratic shocks are more important to firms than monetary shocks).

MR's staggered information-updating assumes that each firm can change its price every period at no cost, but each period it faces a constant probability γ of being given updated information exogenously (between two such opportunities, prices follow the old plan based on outdated information[16]). One possible implicit story behind this assumption is that firms receive information randomly[17] or that they have to make a report on the economic situation at random points in time (for reasons not connected to price setting) and may then use this information for the next time they set prices.

My hybrid process is based upon the assumption that each period a firm faces a probability λ of being given an opportunity to change its price and a probability γ of being given updated information. These two events are assumed to be independent (this is the case if opportunities to set prices

[16]MR assume that, between two re-optimizations, a firm does not know or does not take account of such information as how much it has sold.

[17]For example Carroll (2003) assumes that in any given period each individual faces a constant probability of reading the latest forecast in an article (individuals who do not encounter an article about inflation simply continue to believe the last forecast they read).

are determined by random idiosyncratic shocks, news arrives randomly, and these two random processes are independent).[18] As in the sticky-price model, a firm keeps its price constant between two opportunities to change prices. Firms remember past information when they are given the opportunity to reset prices, and so choose a price based on their last-updated information.

This hybrid model encompasses the two pure models. It yields the sticky-price model if $\gamma = 1$, and yields the sticky-information model if $\lambda = 1$. The possibility that λ be different from 1 is the only difference between my hybrid model and MR's sticky-information model since, as discussed in section 3, I otherwise preserve their economic environment (except for focusing on the strategic-neutrality case in section 5). Since I have assumed that the two pure processes are independent, any interaction between these processes will come only from the fact that a firm can respond to a shock in the hybrid model only if it is both aware of the shock and has an opportunity to change its prices after the shock has occurred.

2.1 Probability that a current price was set j periods ago based on information last updated $j+k$ periods ago

Let $\Omega_{j,k}$ be the probability that the price of a firm at time t was set at $t-j$ (and stayed constant since then) based on information last updated at $t-j-k$. The firm faced a probability λ of being able to change its price at $t-j$, a probability $(1-\lambda)^j$ of not being able to change its prices during the j periods until t. Thus, the probability that at t a firm had its last opportunity to change its price at $t-j$ is $\lambda\,(1-\lambda)^j$. Similarly, the probability that the information available at $t-j$ was last updated at $t-j-k$ is equal to $\gamma\,(1-\gamma)^k$: the probability γ of updating information at $t-j-k$ times the probability $(1-\gamma)^k$ of not being able to update information during the k periods until $t-j$. Since both processes are independent, the probability that the current price of a firm was set j periods ago based on information last updated $j+k$ periods ago is:

$$\Omega_{j,k} = \lambda\gamma\,(1-\lambda)^j\,(1-\gamma)^k \ . \tag{2.1}$$

[18] Here I do not consider the case in which price setting and information updating are state dependent. In this case, the two processes may not be independent. For example, if firms choose to always update their information when they have an opportunity to change their prices, then the hybrid model would in fact be the same as the sticky-price model.

Because of the law of large numbers, $\Omega_{j,k}$ is also the proportion of firms in this situation.

For a time unit of a quarter, MR choose $\lambda = 0.25$ for the sticky-price model, and $\gamma = 0.25$ for the sticky-information model, because in each case a firm on average makes an adjustment once a year. How should λ and γ be chosen in the hybrid model? One possibility is $\lambda = 0.25$ and $\gamma = 0.25$. This, however, implies that the duration between a shock and the first informed price-adjustment is more than one year since overall nominal rigidities have been increased. Let's compute under which condition on λ and γ the average adjustment interval is equal to one year.

2.2 Lag between a shock and the first informed price-adjustment

Let's compute the probability Ξ_t that a firm sets its first informed price-adjustment t periods after the shock (the nature of this shock is not important here, since the price response is not computed in this section). In some particular settings (to be discussed below), this probability will be of particular importance since the inflation response will be proportional to it. Let's assume that an unanticipated monetary shock occurs at the beginning of the period $t = 0$, with some firms possibly already informed of this shock at $t = 0$ before setting their prices (or equivalently, the shock occurs at the end of period $t = -1$ when firms have already set their prices for period $t = -1$): at time $t = 0$ (and possibly later on as well) money supply differs from what all the firms previously expected. A firm that set its first informed price adjustment at t may have been first informed of the shock at $t - j$ (the probability that this happens is $\gamma (1-\gamma)^{t-j}$), and then had to wait j periods to receive an opportunity to change its price (the probability that this happens is $\lambda (1-\lambda)^{j}$). Thus, the probability that a firm sets its first informed price adjustment at t while having been first informed of the shock at $t - j$ is $\lambda\gamma (1-\lambda)^{j} (1-\gamma)^{t-j} = \Omega_{j,t-j}$. Since j could be anywhere between 0 and t, the probability Ξ_t that a given firm sets its first informed price adjustment at t is the sum over $0 \leq j \leq t$ of $\Omega_{j,t-j}$:

$$\Xi_t = \sum_{j=0}^{t} \Omega_{j,t-j} = \sum_{j=0}^{t} \lambda\gamma (1-\lambda)^{j} (1-\gamma)^{t-j} = \begin{cases} \lambda\gamma \frac{(1-\lambda)^{t+1} - (1-\gamma)^{t+1}}{\gamma - \lambda} & \text{if } \lambda \neq \gamma \\ \lambda^2 (t+1)(1-\lambda)^t & \text{if } \lambda = \gamma \end{cases} . \tag{2.2}$$

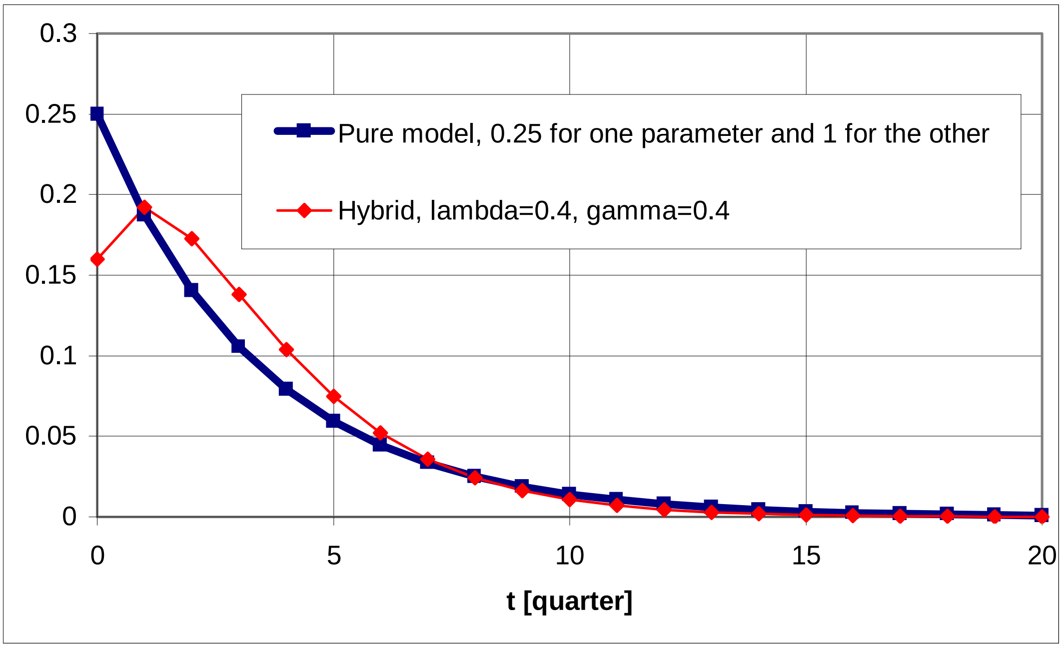
Except for the pure cases $\lambda = 1$ or $\gamma = 1$, the probability Ξ_t is a hump-shaped function of t. The maximum occurs at:

$$t_{\max} = \begin{cases} \frac{\ln\left(\frac{\ln(1-\lambda)}{\ln(1-\gamma)}\right)}{\ln\left(\frac{1-\gamma}{1-\lambda}\right)} - 1 \text{ if } \lambda \neq \gamma \text{ and } \lambda \neq 1 \neq \gamma \\ -\frac{1}{\ln(1-\lambda)} - 1 \qquad \text{if } \lambda = \gamma \\ 0 \qquad \text{if } \lambda = 1 \text{ or } \gamma = 1 \end{cases} . \tag{2.3}$$

The expectation of the distribution Ξ_t is $\frac{1}{\lambda} + \frac{1}{\gamma} - 2$. Taking into account that it is possible to have an informed price-adjustment at $t = 0$, a first informed adjustment at t implies a duration before the first informed adjustment of $t + 1$. Thus, the average duration is $\frac{1}{\lambda} + \frac{1}{\gamma} - 1$. In the sticky-price model, $\gamma = 1$ and this average duration is $\frac{1}{\lambda}$ (and $\lambda = 0.25$ yields an average duration of four quarters). Similarly, this duration is $\frac{1}{\gamma}$ in the sticky-information model (and $\gamma = 0.25$ yields an average duration of four quarters). In the hybrid case, an average duration of four quarters implies the following condition: $\frac{1}{\lambda} + \frac{1}{\gamma} = 5$. Assuming that $\lambda = \gamma$, this yields $\lambda = 0.4 = \gamma$.

For an average duration of four quarters (in the hybrid case $\lambda = \gamma$ is also assumed), Figure 1 shows the distributions of time t of the first informed price-adjustment after a shock occurring at the beginning of period 0.

Figure 1: Distribution of time at which the first informed price adjustment occurs (average duration: 1 year)



Two curves are shown in this figure. The pure sticky-price model and the pure sticky-information model yield the same strictly decreasing curve. The hybrid model with $\lambda = 0.4 = \gamma$ yields a maximum one quarter after the shock.[19]

The distribution for the hybrid curve is hump-shaped (at least for these parameter values), and is thus qualitatively different from the two pure models, which have the same decreasing curve. The intuition is the following. In both pure models the set of firms that have not yet had an informed price-adjustment decreases over time. Since the firms that set their first informed price-adjustment at a given date is a fixed fraction of this set, their number also decreases. In the hybrid model, the firms that set their first informed price-adjustment at a given date were either informed of the shock beforehand, or not even informed. The set of uninformed firms decreases over time. But the set of informed firms that have not yet had the opportunity to change their prices increases, in the hybrid model, at the beginning (at the very beginning it is empty since no firm is informed) and decreases

[19]Notice that even for the hybrid model, the curve may not be hump-shaped (in a discrete-time representation) if this hybrid is sufficiently close to a pure case.

only after having reached a maximum (in the long term, it decreases toward emptiness since the proportion of firms that have not yet had an informed price-adjustment converges toward zero).

3 The economic environment

The last section has only discussed the probability of some events assuming a constant probability λ of receiving an opportunity to change prices and an independent probability γ of updating information. To study the dynamics of inflation, however, it is necessary to know not only when firms change their prices, but also by how much. The price a firm wants to set depends on the economic environment in which the price-setting and information-updating process is imbedded. I follow the simple framework of MR, in which firms set their prices equal to a weighted average of current and future desired prices.

As in MR, I assume:

$$p_t^* = p_t + \alpha y_t \ , \tag{3.1}$$

$$y_t = m_t - p_t \ . \tag{3.2}$$

Equation (3.1) says that a firm's desired price p_t^* depends on the overall price level p_t and the output gap y_t (where all variables are expressed in logs and potential output is normalized to zero). In periods of booms, marginal costs rise and each firm would like to raise its relative price. This equation could be derived from the firm's profit-maximization problem (although the real marginal cost of the firm would appear rather than the output gap) and α could be expressed in terms of deep parameters. Combining equations (3.2) and (3.1) yields $p_t^* = (1-\alpha)\,p_t + \alpha m_t$. Therefore $\alpha = 1$ corresponds to the strategic-neutrality case.

Equation (3.2) expresses aggregate demand as a function of p_t and an exogenous variable m_t, which can be interpreted as the log of money supply or, more broadly, as incorporating the many other variables that shift aggregate demand.[20] More generally, aggregate demand would also depend on the nominal interest rate. Here, however, I follow the simple approach of MR and exclude this possibility.

[20]This equation is used to derive the impulse response function, but not the Phillips curve.

Let $x_{t,k}$ be the price actually set at time t by a firm receiving the opportunity to set its price at time t and holding information updated for the last time at time $t-k$. This firm sets $x_{t,k}$ equal to an average of its expected desired prices for time t and later, weighted by the probability that the price set at time t will not have changed:

$$x_{t,k} = \lambda \sum_{j=0}^{\infty} \left[(1-\lambda)^j E_{t-k} \left(p^*_{t+j} \right) \right], \tag{3.3}$$

where E_{t-k} is the expectation of firms with information last updated at time $t-k$ (or equivalently the expectation of the best-informed firms at $t-k$).

In the sticky-price case, a firm is always informed, thus $k=0$ and equation (3.3) becomes $x_{t,k=0} = \lambda \sum_{j=0}^{\infty} \left[(1-\lambda)^j E_t \left(p^*_{t+j} \right) \right]$. In the sticky-information case, $\lambda = 1$, and equation (3.3) is rewritten as $x_{t,k} = E_{t-k}\left(p^*_t\right)$.[21]

The aggregate price level is the average of prices set by the various cohorts of firms, weighted by the proportion of firms in each cohort:

$$p_t = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \Omega_{j,k} x_{t-j,k} \quad \text{where} \quad \Omega_{j,k} = \lambda\gamma (1-\lambda)^j (1-\gamma)^k \ . \tag{3.4}$$

In the sticky-price case ($\gamma = 1$) equation (3.4) becomes $p_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j x_{t-j,k=0}$.

In the sticky-information case ($\lambda = 1$) equation (3.4) becomes $p_t = \gamma \sum_{k=0}^{\infty} (1-\gamma)^k x_{t,k}$.

4 The Phillips curve

The Phillips curve yields a relationship between prices and the output gap. This is an intermediate stage before computing the inflation impulse response, since plugging equation (3.2) into the Phillips curve yields a relationship between prices and money supply. This section presents the Phillips curve without yet assuming strategic neutrality.

[21] $(1-\lambda)^j$ is, in fact, undetermined when $\lambda = 1$ and $j = 0$. Actually, this is calculated assuming λ infinitesimally close to 1 but not equal to 1. The resulting formula is indeed the same as the one used by Mankiw and Reis (2002) for the sticky-information case. A similar comment applies in other places in this paper.

Appendix I shows that, after some tedious algebra, equations (3.1), (3.3) and (3.4) yield the following Phillips curve:

$$\pi_t - \overline{E}_t(\pi_{t+1}) = \frac{\lambda^2}{1-\lambda}\left[\alpha y_t + \varepsilon_t(p_t^*)\right], \tag{4.1}$$

where ε_t is an operator that takes the sum of expectation errors made at t (i.e. the average of expectation errors made by various cohorts weighted by the number of firms in each cohort). In the hybrid model, the sum of expectation errors made at t on p_t^*, is equal to:

$$\varepsilon_t(p_t^*) = \gamma\sum_{k=1}^{\infty}(1-\gamma)^k\left[E_{t-k}(p_t^*) - p_t^*\right]. \tag{4.2}$$

The term $\overline{E}_t(\pi_{t+1})$ in equation (4.1) is defined as follows:

$$\overline{E}_t(\pi_{t+1}) = \lambda\left[\gamma\sum_{k=0}^{\infty}(1-\gamma)^k x_{t+1,k+1} - p_t\right]. \tag{4.3}$$

The expectation operator $\overline{E}_t$ obviously differs from the expectation E_t established on the basis of the best information available at t (or equivalently, made by the best-informed agents at t), since there is no reason why only the expectations of the best-informed firms should matter while the inflation expectations of firms setting their price at time based on old information would be completely neglected. What is perhaps more surprising is that the relevant expectation operator is not simply an average of the various inflation expectations.[22] Equation (4.3) gives the inflation expected to prevail at time $t+1$ when the aggregate price level at $t+1$ is expected to be $\gamma\sum_{k=0}^{\infty}(1-\gamma)^k x_{t+1,k+1}$ while the aggregate price level at time t is known to be p_t. One interpretation is the following: make a survey asking all firms[23] by how much they expect to increase their own prices from t to $t+1$ (don't ask them about their expectations for the increase of the aggregate price level); then $\overline{E}_t(\pi_{t+1})$ is the sum of these expected price increases. The proof is the following. Each firm will answer that it faces a probability $1-\lambda$ of keeping

[22] The average inflation expectation is $\lambda\left[\gamma\sum_{k=0}^{\infty}(1-\gamma)^k E_{t-k}(p_{t+1}) - p_t\right]$. See section 5 for a specific example in which the operator $\overline{E}_t(\pi_{t+1})$ is shown to be different from the average inflation expectations.

[23] Ask all firms once they know if they can reset their price at time t or not (firms that cannot reset their prices at time t are also to be included in the survey).

its price constant, and a probability λ of being able to reset its price. Thus, its expected increase of its own price is λ times the difference between the price it expects to set if it is able to reset it and its current price (all firms are aware of their current prices). Summing all these answers yields equation (4.3): λ times the difference between the sum $\gamma\sum_{k=0}^{\infty}(1-\gamma)^k x_{t+1,k+1}$ of all answers about the prices firms would expect to set at $t+1$ (if they can) and the sum p_t of their current prices.

It is easy to verify that if $\gamma = 1$, equation (4.1) boils down to the sticky-price Phillips curve $\pi_t - E_t(\pi_{t+1}) = \frac{\lambda^2}{1-\lambda}\alpha y_t$ (given, for example, in MR). In fact, when $\gamma = 1$, then $\varepsilon_t(p_t^*) = 0$ (i.e. there are no expectational errors at t on p_t^* since all firms are informed) and $\overline{E}_t(\pi_{t+1}) = E_t(\pi_{t+1})$ (since all firms have the same information set). In the other pure case, if $\lambda = 1$, then $\overline{E}_t(\pi_{t+1})$ disappears (firms do not need to take account of future inflation when setting their current prices, since they can change their prices in every period), and the Phillips curve is given by $\alpha y_t + \varepsilon_t(p_t^*) = 0$, which can be shown to be equivalent to the Phillips curve computed by MR for the sticky-information model.

The hybrid Phillips curve could be compared with the Phillips curves of other models. Three models would be particularly interesting in this respect. Woodford (2003) assumes that information-updating does not occur with a constant probability but is simply delayed by a fixed number of periods (thus extending a model he wrote with Rotemberg, in which the delay is always one period). Altig et al. (2005) assume that between two re-optimizations firms follow simple (non-optimal) indexation rules. Gali and Gertler (1999) assume that a fraction of firms set prices according to a rule of thumb (they index their prices according to last-period inflation) while the other firms have rational expectations.

Some differences between their Phillips curves and mine are due to a difference in frameworks. But even after adapting their models to MR's framework (this involves setting the preference for the present to zero and assuming that the real marginal cost is proportional to output) important differences remain. In the Rotemberg-Woodford model, when a firm sets its price for a given date, it always perfectly anticipates the aggregate price-level that will prevail at that date because all other firms will be setting their prices for that date on the basis of the same common information set. This is not the case in my hybrid model. An important difference between my hybrid model, on one hand, and the Gali-Gertler model or the model of Altig et al.

(2005), on the other hand, is that their Phillips curves do not involve past expectations whereas my hybrid model does (it inherits this feature from the sticky-information model).

5 The inflation impulse response in the strategic-neutrality case

Strategic neutrality in price setting means that a firm's desired price does not depend on the prices set by competitors. After examining the relevance of this assumption, I discuss the inflation impulse response to three types of unanticipated shocks: i) a transitory shock to the money-supply growth rate or, equivalently, a permanent shock to the money-supply level (which is experiment 1 in MR), ii) a permanent shock to the money-supply growth rate (experiment 2 in MR), and iii) the intermediate case of a persistent but not permanent shock to the money-supply growth rate.

5.1 Strategic neutrality

How can the desired price of a firm be independent of the prices set by competitors? A first answer would be that in the standard monopolistic-competition model à la Dixit-Stiglitz, firms set their prices at a constant markup over the marginal cost. Thus the prices of competitors do not directly influence the desired price. This answer, however, does not take into account the possibility that the prices of competitors may indirectly influence the desired price, through their impact on marginal costs.

During the staggered price-setting process, a firm that adjusts its prices is motivated by several incentives. First, it might want to adjust less than what it would if all other firms had the opportunity to adjust, because it faces competition from those firms that have not adjusted yet. This strategic complementarity in price setting may hold even if firms simply choose a markup that is a constant proportion of the marginal cost without taking prices set by competitors directly into account. For example, assume there is a reduction in money supply. Then a firm that adjusts its prices downward, while some other firms have not yet done so, will face greater demand than otherwise. If marginal productivity decreases with output (or if there are some firm-specific factors), then marginal costs increase with output (assuming that the prices of production factors bought on the economy-wide market stay constant), and, even if the firm faced no nominal rigidities, it would want to set its price higher than it would if other firms didn't face nominal rigidities in order not to be overburdened by a overly high demand.

However, the price of production factors bought on the economy-wide market need not stay constant. This can lead the firm to decrease its prices by more than needed to reach the frictionless equilibrium (strategic substitutability in price setting). Assuming that the real wage is pro-cyclical, it will be lower during the transitory recession generated by the reduction in money supply. This tends to decrease marginal cost and thus lead to lower prices. If the complementarity and the substitutability incentives cancel each other out, there is strategic neutrality in price setting: a firm would choose to set its prices independently of the aggregate price level.

Whether there is complementarity or substitutability in price-setting is a much debated issue in the literature. Using a dynamic stochastic general equilibrium (DSGE) model with sticky prices, Chari et al. (2000) find that strategic substitutability arises from realistic deep-parameter values of their model. On the other hand, Woodford (2003) argues that Chari and al. would find strategic complementarity in firms' price decisions if they had taken into account the existence of firm-specific production factors. Incorporating sticky information into different DSGE models, Keen (2005) finds strategic substitutability, whereas Trabandt (2006) finds strategic complementarity. My reading of the current state of this debate is that it is not settled yet, and that the middle ground of assuming strategic neutrality would be compatible with the literature.

5.2 Permanent shocks to the money-supply level

Let's consider the case of a shock δ to the level of money supply. Appendix II shows that in this case, for $t \geq 0$, the aggregate price level p_t in the hybrid model is given by:

$$\begin{aligned} p_t &= m_t + \delta \left[\frac{\lambda}{\gamma - \lambda} (1-\gamma)^{t+2} - \frac{\gamma}{\gamma - \lambda} (1-\lambda)^{t+2} \right] \quad \text{for } \lambda \neq \gamma \qquad (6.1) \\ p_t &= m_t - \delta (1-\lambda)^{t+1} \left[1 + (t+1)\lambda \right] \quad \text{for } \lambda = \gamma \, . \end{aligned}$$

Whereas, in the sticky-price and the sticky-information models, p_t is respectively given by:

$$\begin{aligned} p_{t,\gamma=1} &= m_t - \delta (1-\lambda)^{t+1} \qquad \text{for } \gamma = 1 \\ p_{t,\lambda=1} &= m_t - \delta (1-\gamma)^{t+1} \qquad \text{for } \lambda = 1. \end{aligned}$$

Thus the pure sticky-price and the pure sticky-information cases yield the same price dynamics if they are calibrated to get the same average duration between two re-optimizations, that is, if the parameter λ used in the sticky-price case is the same as the parameter γ used in the sticky-information case.

If $\lambda \neq \gamma$, the price level in the hybrid case p_t can be expressed as a linear combination of the two pure cases' price levels:
$p_t = \frac{\gamma(1-\lambda)p_{t,\gamma=1} - \lambda(1-\gamma)p_{t,\lambda=1}}{\gamma-\lambda}$ where the weights $\frac{\gamma(1-\lambda)}{\gamma-\lambda}$ and $-\frac{\lambda(1-\gamma)}{\gamma-\lambda}$ have opposite signs (thus, it is a linear combination, but not a weighted average although the sum is 1). Or equivalently: $p_t = p_{t,\gamma=1} + \lambda\frac{1-\gamma}{\lambda-\gamma}\left(p_{t,\lambda=1} - p_{t,\gamma=1}\right)$.

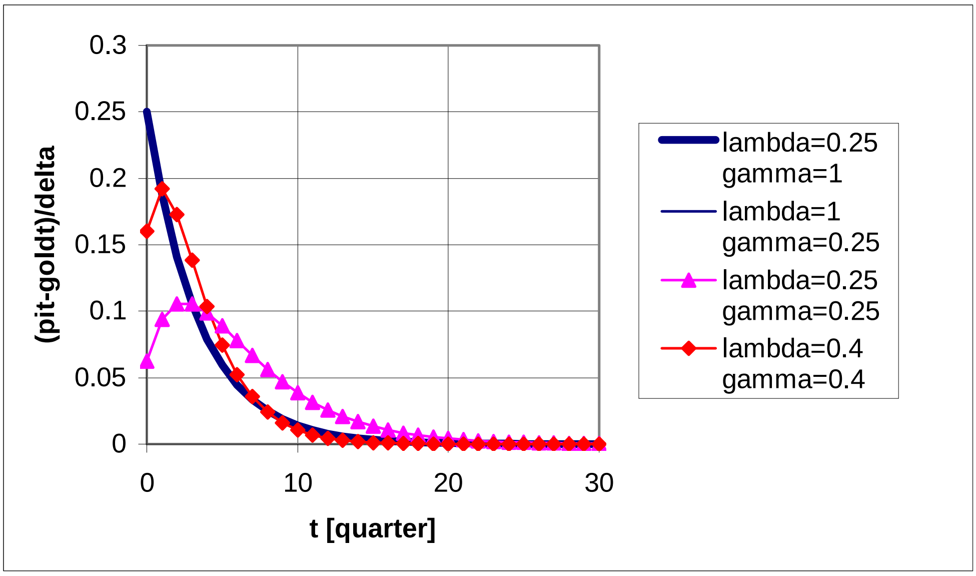
Equation (6.1) implies that the inflation response for $t \geq 0$ is given by:

$$\pi_t - g = \begin{cases} \delta\ \Xi_t & \text{for } \ t \geq 0 \\ 0 & \text{for } \ t < 0 \end{cases} \quad , \tag{6.2}$$

where g is the money-supply growth rate for $t \neq 0$.

The inflation response after the shock is proportional to the probability Ξ_t given in equation (2.2) that a firm sets its first informed price-adjustment at t. Thus Figure 1 already shows the impulse response of inflation for different configurations of the parameters representing the degree of price and information stickiness. This figure is reproduced below (a curve giving the impulse response for $\lambda = 0.25 = \gamma$ is added). As discussed in section 2, setting λ and γ equal to 0.4 yields the same average duration before the first informed adjustment as in the sticky-price model ($\lambda = 0.25$ & $\gamma = 1$) or the sticky-information model ($\lambda = 1$ & $\gamma = 0.25$). On the other hand, the hybrid model calibrated at $\lambda = 0.25 = \gamma$ generates a greater degree of nominal rigidity.

Figure 2: Inflation impulse response to a permanent shock to the money-supply level



With these parameter values, the sticky-information model yields the same inflation response as the sticky-price model. Moreover, after the initial jump, it decreases monotonically, whereas the impulse response in the hybrid model is hump-shaped. When for the hybrid model the probability of being informed is the same as the probability of having the opportunity to reset prices, and probabilities are calibrated as in MR to yield an average duration of one year before an informed price-adjustment takes place, then the maximum occurs one quarter after the shock. If the overall nominal rigidities are larger ($\lambda = 0.25 = \gamma$), then the maximum response of inflation occurs later.

The intuition as to why inflation is proportional to the probability of setting a first informed price-adjustment is easiest to understand if the level of money supply is constant before the shock, is modified by the shock, and remains at its new level after the shock. In this scenario, the price set after the shock by an informed firm is at its long-term equilibrium. In this case firms change their prices only once: they adjust to the long-term equilibrium as soon as they can set their first informed price-adjustments. Since all firms had the same price before the shock and end up with the same new long-term equilibrium, the inflation response is proportional to the number of first informed price-adjustments.

The impulse response of inflation computed for the special case of a zero money-supply growth rate remains the same for any other constant growth rate. To see this, consider equation (II.2):

$$p_t = (1-\lambda)\, p_{t-1} + \lambda^2 \gamma \sum_{k=0}^{\infty} (1-\gamma)^k \left[\sum_{j=0}^{\infty} \left[(1-\lambda)^j\, E_{t-k}\left((1-\alpha)\, p_{t+j} + \alpha m_{t+j} \right) \right] \right].$$

If the money-supply dynamics are of the form $\widetilde{\widetilde{m}}_t = \widetilde{\widetilde{m}}_{-1} + \widetilde{\widetilde{g}}\,(t+1)$, then $p_t = m_t$ in the absence of shocks. Therefore, equation (II.2) can be rewritten replacing p by $\widetilde{p} = p - \widetilde{\widetilde{m}}$ and m by $\widetilde{m} = m - \widetilde{\widetilde{m}}$. Choosing $\widetilde{\widetilde{m}}_{-1} = m_{-1}$ and $\widetilde{\widetilde{g}} = g$, it follows that $\widetilde{\widetilde{m}}_t = m_{new,t}$ (where $m_{new,t}$ extends to all time the affine relationship between t and m that holds after the shock), and thus $\widetilde{m}_t = 0$ after the shock.

If we know how to solve (II.2) for the price dynamics in the case where money supply is constant after the shock, and want to know the price dynamics when money supply grows at a constant rate after the shock, all we need to do is subtract $\widetilde{m}_t = m_{new,t}$ from the p and m variables to be in the setting in which money supply is zero after the shock, solve for the dynamics of $\widetilde{p}$, and add back $m_{new,t}$. Adding back $m_{new,t}$ will change the aggregate price level but not the difference between inflation with the shock and inflation without the shock (the constant $\widetilde{\widetilde{m}}_{-1}$ disappears when inflation is computed, and the term $\widetilde{\widetilde{g}}$ disappears when the difference between inflation with and without the shock is computed). The point is that, and this is true in the general case since the argument is based on equation (II.2), the inflation impulse response is invariant to a transformation of the money-supply dynamics consisting in adding a money-supply component with a constant growth rate.

Let's compute $\overline{E}_t\left(\pi_{t+1}\right)$ for $t = 0$ in the simple case where the level of money supply is 0 before the shock, is modified to δ by the shock, and remains at δ after the shock. Firms unaware of the shock expect to keep their prices constant even if they are allowed to reset them. Similarly, firms aware of the shock, and having had the opportunity to make an informed price adjustment, have fully adjusted already, and thus expect to keep their prices constant even if they could reset prices in the future (expectations are polled at a point in the period $t = 0$ when each firm knows if it can reset its price at $t = 0$ or not). The only firms expecting to change their own prices are those that are informed of the shock but haven't had an opportunity for an informed price adjustment. These firms, which at $t = 0$ are a proportion $\gamma\,(1-\lambda)$ of all firms, expect to increase their own prices by an amount δ if they have an opportunity to do so. Thus, $\overline{E}_0(\pi_1) = \gamma(1-\lambda)\lambda\delta$: the probability $\gamma(1-\lambda)$ of being informed without having had an opportunity for an informed price-adjustment times the probability λ of having an opportunity to adjust prices next period times the price change δ. The average inflation expectations can also be computed. Firms unaware of the shock expect zero inflation. Firms aware of the shock (whether they had an opportunity to make an informed price adjustment or not), which at $t=0$ are a proportion γ of all firms, know that inflation at time $t=1$ will be $\delta\ \Xi_1$. Thus, the average next period inflation expected at $t=0$ is $\gamma\delta\Xi_1$. From equation (2.2), $\Xi_1 = 2\lambda\gamma\left(1-\frac{\lambda+\gamma}{2}\right)$. Hence, the ratio at $t=0$ of average inflation expectations to $\overline{E}_0(\pi_1)$ is $2\gamma\frac{1-\frac{\lambda+\gamma}{2}}{1-\lambda}$. This ratio is equal to 1 if $\gamma=1$, i.e. in the pure sticky-price case. If $\gamma \neq 1$, this ratio is usually different from 1 (the only other exception is when $\gamma = 1-\lambda$). This example proves that the operator $\overline{E}$ can be different from the average expectations.

5.3 Permanent shocks to the money-supply growth rate

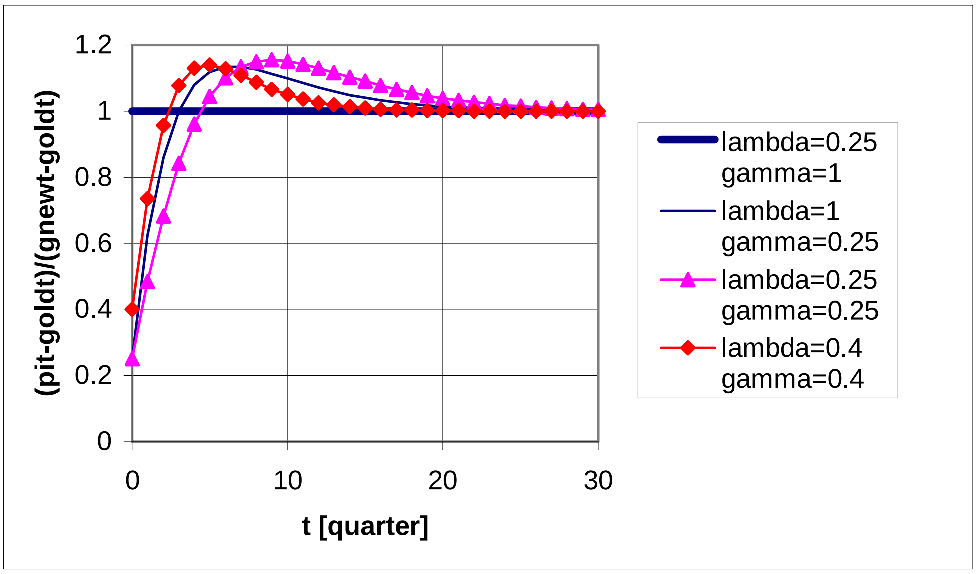
Let's continue to assume that firms are strategically neutral, but that at time $t=0$ there is a shock to the money-supply growth rate rather than to the level of money supply. Equation (II.13) in Appendix II implies that in this case the level of inflation for $t \geq 0$ is given by:

$$\begin{aligned}
\pi_t &= g + (g - g_{old}) \begin{bmatrix} -t\frac{\lambda\gamma(1-\gamma)^{t+1}}{\gamma-\lambda} \\ +\frac{\lambda\gamma(1-\gamma)(1-\lambda)\left[(1-\lambda)^t-(1-\gamma)^t\right]}{(\gamma-\lambda)^2} \\ -(1-\gamma)^{t+1} \end{bmatrix} \text{ for } \lambda \neq \gamma \qquad (6.3) \\
\pi_t &= g + (g - g_{old})(1-\lambda)^t \left[\frac{\lambda^2}{2}t(t+1) - (1-\lambda)\right] \text{ for } \lambda = \gamma.
\end{aligned}$$

Notice that (for $t \geq 0$):
If $\gamma = 1$ then $\pi_t = g$.
If $\lambda = 1$ then $\pi_t = g + \left[-(1-\gamma)^{t+1} + \gamma(1-\gamma)^t t\right](g - g_{old})$.
If $\lambda = \gamma$ then $\pi_t = g + \left[-(1-\lambda)^{t+1} + \lambda^2(1-\lambda)^t t(t+1)/2\right](g - g_{old})$.

Figure 3 shows the impulse response to a money-supply growth shock, using the same numerical example as for Figure 2.

Figure 3: Inflation impulse response to a permanent shock to the money-supply growth rate



These curves show the impulse response at time t as a proportion of the long-run response. All curves converge toward 1 in the long run. For the sticky-price case ($\gamma = 1$) the inflation response is flat, whereas it is hump-shaped for the sticky-information case ($\lambda = 1$), reaching a maximum at $t = \frac{1-\gamma}{\gamma} - \frac{1}{\ln(1-\gamma)}$. The impulse response is also hump-shaped for the hybrid case both when $\lambda = \gamma = 0.25$ and when $\lambda = \gamma = 0.4$. The two hybrid curves are hump-shaped, but the maximum occurs later for the first curve. There is a jump in inflation at time $t = 0$ for all four curves, but this jump is much larger for the sticky-price curve than for the other curves. Overall, the two hybrid curves have the same qualitative features as the sticky-information curve, both contrasting sharply with the sticky-price curve. In contrast with the case of a shock to the money-supply level, the sticky-information curve is similar to the hybrid case here. The advantage of the hybrid curve relates to micro evidence rather than macro evidence: in the sticky-information model every firm changes its price every period (in contrast with micro evidence), whereas this is not the case in the hybrid model.

5.4 Persistent but not permanent shocks to the money-supply growth rate

Let's consider another type of monetary shock. Suppose that

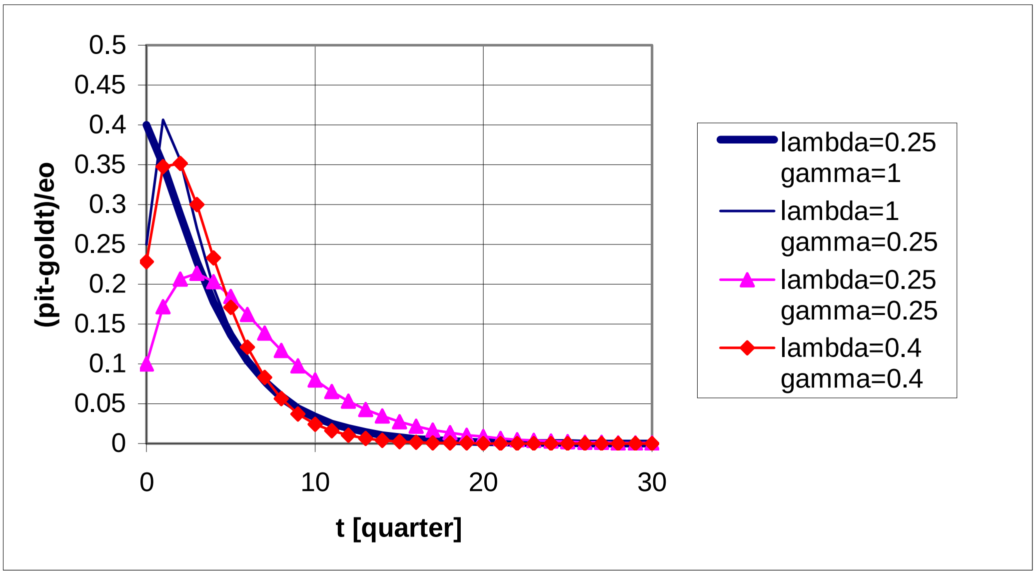
$$m_{old,t} = m_{-1} + (t+1)\, g_{old} \text{ for } t < 0 \ ,$$

$$m_t = m_{-1} + (t+1)\, g_{old} + \varepsilon_0 \sum_{i=0}^{t} \rho^i \text{ for } t \geq 0 \ ,$$

where ρ is the autocorrelation of the money-supply growth rate. If $\rho = 0$, then the shock is completely transitory, corresponding to the permanent shock to the level of money supply discussed in section 5.2. If $\rho = 1$, then the shock is permanent, corresponding to the permanent shock to the money-supply growth rate discussed in section 5.3. Intermediate values of ρ correspond to a persistent but not permanent shock to the money-supply growth rate: the money-supply growth rate changes the most at $t = 0$, then changes each period by a smaller amount and converges toward its initial value g_{old}. During this process, a change of the money-supply level of $\frac{\varepsilon_0}{1-\rho}$ builds up. In considering this experiment, MR regard the value of $\rho = 0.5$ as being realistic for U.S. quarterly data.

In the strategic-neutrality case, it is possible to find a closed-form solution for the aggregate price dynamics (see equation II.14 in appendix II). Figure 4 shows the impulse response of inflation using the same numerical example as for Figures 2 and 3.

Figure 4: Inflation impulse response to a shock to the money-supply growth rate with autocorelation $\rho = 0.5$



This figure shows that the inflation impulse response is not hump-shaped in the sticky-price model whereas it is hump-shaped in the sticky-information model and in the hybrid model. The inflation impulse responses of the sticky-information and the hybrid models are still qualitatively similar. Section 5.2 has shown that decreasing the persistence of the shock ultimately favors the hybrid model. For the intermediate case of $\rho = 0.5$, a small advantage for the hybrid model can already be observed in the sense that the inflation jump at the shock is smaller and the hump appears later in the hybrid model (the exact date at which the hump appears depends on the calibration).

6 Conclusion

Most macroeconomic models cannot explain the two following stylized facts simultaneously: i) individual firms change prices every six months to a year and ii) shocks to monetary policy have a delayed and gradual effect on inflation. This paper presents a hybrid sticky-price and sticky-information model compatible with both of these facts.

In the case of a permanent shock to the growth rate of money supply, the inflation responses are hump-shaped both in the hybrid and in the sticky-information models, and both are clearly different from the monotonic response of the sticky-price model. Reducing the persistence of the shock ultimately favors the hybrid model. If the shock is completely transitory, the hybrid model delivers a hump-shaped inflation response to monetary policy, whereas the two pure cases yield the same strictly decreasing response. Intuitively, this result relates to the hump-shaped dynamics of the number of informed firms that have not yet had the opportunity to re-optimize their prices since the shock occurred. In the intermediate case of a shock to the money-supply growth rate with an autocorrelation coefficient equal to 0.5, the hybrid and the sticky-information models yield qualitatively similar inflation impulse responses.

On the way to computing inflation impulse responses, I derive the Phillips curve for the hybrid model (without, at this stage, assuming anything about strategic neutrality). One novel feature of this Phillips curve is that it involves a new kind of expectation operator.

This study could be extended by considering other kinds of monetary shocks or deviation from strategic neutrality. It is not clear that in these settings the hybrid model will be so distinctly superior to the pure models as it is in the examples discussed here. A priori, it could be expected that the hybrid model behaves more as an average of the pure models would in the case of anticipated shocks. Whether this is the case or not could be checked.

The results of this paper have been obtained under the assumption of strategic neutrality in the pricing decisions of firms. It would be important to discuss the case of strategic complementarity since this may increase persistence. Would strategic complementarity penalize the hybrid model relative to the sticky-information model? Strategic complementarity is likely to make the sticky-information inflation impulse-response more hump-shaped. It would be interesting to know how the inflation impulse response changes according to the degree of strategic complementarity in the hybrid model. Another issue is that strategic complementarity may allow the hybrid model to remedy a shortcoming of the sticky-information model that has not yet been discussed in this paper: in the sticky-information model, a monetary shock would have no impact on inflation once all firms are informed of the shock.[24] This would be obvious if information updating were assumed to occur at intervals of constant duration (then all firms would be informed after that duration) rather than with a constant probability (in which case there

[24] Collard and Dellas (2003) and Dupor and Tsuruga (2005) criticize the sticky-information model on this account.

are always some firms that are not informed yet). But a hybrid of the standard sticky-price model and a sticky-information model in which information is updated at constant intervals would become a sticky-price model as soon as all firms are informed. Thus, the hybrid model may inherit the endogenous persistence that, assuming strategic complementarity, the sticky-price model features in most cases.[25]

Other interesting further research would include: i) discussing the case in which the price-adjustment opportunity and information updating are not independent or are state-dependent, and ii) extending the model to a general-equilibrium setting.

[25] This phenomenon is best thought of as a "contract multiplier," as Taylor (1980) put it. In the special case where perfect adjustment is immediate (this happens in the case of a permanent shock to the money-supply growth rate), the contract multiplier cannot generate additional persistence.

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Appendix I: Derivation of the hybrid Phillips curve

Two intermediate equations

Let's first derive two equations that will be useful later.

Equation (3.3) yields:

$$x_{t,k} = \lambda E_{t-k}\left(p_t^*\right) + (1-\lambda)\, x_{t+1,k+1} \tag{I.1}$$

because $x_{t,k} = \lambda \sum_{j=0}^{\infty} \left[(1-\lambda)^j E_{t-k}\left(p_{t+j}^*\right)\right] = \lambda E_{t-k}\left(p_t^*\right) + \lambda \sum_{j=1}^{\infty} \left[(1-\lambda)^j E_{t-k}\left(p_{t+j}^*\right)\right]$
$= \lambda E_{t-k}\left(p_t^*\right) + (1-\lambda)\, \lambda \sum_{j=0}^{\infty} \left[(1-\lambda)^j E_{t-k}\left(p_{t+1+j}^*\right)\right]$
$= \lambda E_{t-k}\left(p_t^*\right) + (1-\lambda)\, x_{t+1,k+1}$.

Equation (3.4) yields:

$$p_t = \lambda\gamma \sum_{k=0}^{\infty} \left[(1-\gamma)^k x_{t,k}\right] + (1-\lambda)\, p_{t-1} \tag{I.2}$$

because

$p_t = \lambda\gamma \sum_{j=0}^{\infty} \left[(1-\lambda)^j \sum_{k=0}^{\infty} (1-\gamma)^k x_{t-j,k}\right]$
$= \lambda\gamma \left[\sum_{k=0}^{\infty} (1-\gamma)^k x_{t,k}\right] + \lambda\gamma \sum_{j=1}^{\infty} \left[(1-\lambda)^j \sum_{k=0}^{\infty} (1-\gamma)^k x_{t-j,k}\right]$,
where
$\lambda\gamma \sum_{j=1}^{\infty} \left[(1-\lambda)^j \sum_{k=0}^{\infty} (1-\gamma)^k x_{t-j,k}\right]$
$= (1-\lambda)\, \lambda\gamma \sum_{j=0}^{\infty} \left[(1-\lambda)^j \sum_{k=0}^{\infty} (1-\gamma)^k x_{t-1-j,k}\right]$
$= (1-\lambda)\, p_{t-1}$

Rewriting the hybrid Phillips curve

The hybrid Phillips curve $\pi_t - \overline{E}_t(\pi_{t+1}) = \frac{\lambda^2}{1-\lambda}\left[\alpha y_t + \varepsilon_t(p_t^*)\right]$,
where $\overline{E}_t(\pi_{t+1}) = \lambda\left[\gamma\sum_{k=0}^{\infty}(1-\gamma)^k x_{t+1,k+1} - p_t\right]$
and $\varepsilon_t(p_t^*) = \gamma\sum_{k=1}^{\infty}(1-\gamma)^k\left[E_{t-k}(p_t^*) - p_t^*\right]$,
can be rewritten as:
$\pi_t - \lambda\left[\gamma\sum_{k=0}^{\infty}(1-\gamma)^k x_{t+1,k+1} - p_t\right] = \frac{\lambda^2}{1-\lambda}\left[\alpha y_t + \gamma\sum_{k=1}^{\infty}(1-\gamma)^k\left[E_{t-k}(p_t^*) - p_t^*\right]\right]$,
which, using $p_t^* = p_t + \alpha y_t$ and $\pi_t = p_t - p_{t-1}$, becomes after some algebraic transformations:

$$\frac{p_t - (1-\lambda)p_{t-1} - (1-\lambda)\gamma\sum_{k=0}^{\infty}\left[(1-\gamma)^k\lambda x_{t+1,k+1}\right]}{\lambda^2} = \gamma\sum_{k=0}^{\infty}(1-\gamma)^k E_{t-k}(p_t + \alpha y_t). \tag{I.3}$$

Deriving the hybrid Phillips Curve

Equation (I.3) is true since

$$\frac{p_t - (1-\lambda)p_{t-1} - (1-\lambda)\gamma\sum_{k=0}^{\infty}\left[(1-\gamma)^k\lambda x_{t+1,k+1}\right]}{\lambda^2}$$

$$= \frac{\lambda\gamma\sum_{k=0}^{\infty}\left[(1-\gamma)^k x_{t,k}\right] - (1-\lambda)\gamma\sum_{k=0}^{\infty}\left[(1-\gamma)^k\lambda x_{t+1,k+1}\right]}{\lambda^2} \qquad \text{using equation (I.2)}$$

$$= \frac{\lambda\gamma\sum_{k=0}^{\infty}\left[(1-\gamma)^k\left[\lambda E_{t-k}(p_t^*) + (1-\lambda)x_{t+1,k+1}\right]\right] - (1-\lambda)\gamma\sum_{k=0}^{\infty}\left[(1-\gamma)^k\lambda x_{t+1,k+1}\right]}{\lambda^2} \qquad \text{using equation (I.1)}$$

$$= \gamma\sum_{k=0}^{\infty}\left[(1-\gamma)^k E_{t-k}(p_t^*)\right] \qquad \text{algebra}$$

$$= \gamma\sum_{k=0}^{\infty}(1-\gamma)^k E_{t-k}(p_t + \alpha y_t) \qquad \text{using equation (3.1)}$$

Appendix II: The inflation impulse response

This appendix gives some equations for computing the impulse responses and explains why computation of the impulse response is easier in the pure cases (sticky prices or sticky information) and in the strategic neutrality case than in the general case.

II.1) The general case

Plugging equations (3.1), (3.2), (3.3), (4.2) and (4.3) into the hybrid Phillips curve (4.1) yields:

$$\begin{aligned} & (1-\lambda)\,\lambda\gamma\sum_{k=0}^{\infty}(1-\gamma)^k\,x_{t+1,k+1} \\ & -\left[1-\lambda^2\gamma\,(1-\alpha)\right]p_t+(1-\lambda)\,p_{t-1} \\ = \; & -\lambda^2\gamma\left\{\alpha m_t+\sum_{k=1}^{\infty}(1-\gamma)^k\left[E_{t-k}\left((1-\alpha)\,p_t+\alpha m_t\right)\right]\right\}. \end{aligned} \tag{II.1}$$

Plugging into equation (II.1) $x_{t+1,k+1}$ expressed in terms of p and m according to equation (3.3) yields:[26]

$$p_t=(1-\lambda)\,p_{t-1}+\lambda^2\gamma\sum_{k=0}^{\infty}(1-\gamma)^k\left[\sum_{j=0}^{\infty}\left[(1-\lambda)^j\,E_{t-k}\left((1-\alpha)\,p_{t+j}+\alpha m_{t+j}\right)\right]\right]. \tag{II.2}$$

In the general case this equation involves an infinity of aggregate prices (all aggregate prices from $t-1$ and thereafter). Some algebraic transformations can, however, reduce this dimensionality to a third-order recursive equation with variable coefficients. This equation is solvable (not necessarily analytically, but at least numerically). The resolution of the general case is left for further research. I will focus below on three special cases in which computation is simple.

II.2) Three simple cases

Sticky information

In the sticky-information case, $\lambda=1$, and the future aggregate prices on the right-hand side of equation (II.2) disappear (and so does the last-period aggregate price), which yields a simple equation in p_t:

[26] Alternatively, plug equations (3.1), (3.2) and (3.3) into equation (I.2) of appendix I.

$$p_t = \gamma \sum_{k=0}^{\infty} (1-\gamma)^k \left[E_{t-k} \left((1-\alpha) p_t + \alpha m_t \right) \right]. \tag{II.3}$$

This equation can be solved (assuming rational expectations) once the nature of the monetary shock is specified.

In the absence of shocks, $E_{t-k}(p_t) = p_t$ and $E_{t-k}(m_t) = m_t$, and equation (II.3) yields $p_t = m_t$ (whatever the dynamics of m).

If there is only a lone shock occurring at time $t = 0$, and if it is not anticipated, then (II.3) becomes:

$$\frac{p_t - m_t}{m_{old,t} - m_t} = \frac{(1-\gamma)^{t+1}}{\alpha + (1-\gamma)^{t+1}(1-\alpha)}, \tag{II.4}$$

where $m_{old,t}$ is the actual money-supply before the shock, or the money supply that would have prevailed after time $t = 0$ if no shock had occurred. m_t still denotes the actual money-supply ($m_t = m_{old,t}$ for $t < 0$, but is different from $m_{old,t}$ at $t = 0$, and may be different later on as well). Equation (II.4) gives a measure of the incompleteness of the price adjustment made at time t (as a proportion of the adjustment that would have been made at time t in the absence of nominal rigidities). For $t < 0$, equation (II.4) yields $p_t = m_t$. Inflation can be computed by extracting p_t from equation (II.4) and substracting a lagged version of this equation.

Sticky prices

In the pure sticky-price model, equation (II.2) becomes (after plugging in $\gamma = 1$):

$$p_t = (1-\lambda) p_{t-1} + \lambda^2 \sum_{j=0}^{\infty} \left[(1-\lambda)^j E_t \left((1-\alpha) p_{t+j} + \alpha m_{t+j} \right) \right]. \tag{II.5}$$

The future aggregate price levels are still present, but they can be eliminated easily by writing equation (II.5) for $t+1$, taking the expectation at t, and substracting equation (II.5) for t divided by $1-\lambda$. This yields:

$$E_t p_{t+1} - \left(2 + \frac{\alpha \lambda^2}{1-\lambda} \right) p_t + p_{t-1} = -\frac{\alpha \lambda^2}{1-\lambda} m_t. \tag{II.6}$$

This is a second-order recursive equation with constant coefficients. There are several ways to solve this equation analytically. For example, the dimensionality can be further reduced to

$$p_t = \theta p_{t-1} + (1-\theta)^2 \sum_{i=0}^{\infty} \theta^i E_t m_{t+i}, \tag{II.7}$$

where θ is solution of $2 + \frac{\alpha\lambda^2}{1-\lambda} = \theta + \frac{1}{\theta}$ such that $\theta < 1$. The solution of (II.7) is:

$$p_t = (1-\theta)^2 \sum_{k=0}^{\infty}\sum_{i=0}^{\infty} \theta^{i+k} E_{t-k} m_{t-k+i}. \tag{II.8}$$

Assuming that there is only a lone unanticipated shock occurring at time $t = 0$, equation (II.8) could be expressed in terms of $m_{old,t}$ and m_t. In the sticky-price model it particularly makes sense to focus on cases in which the growth rate of the money supply (without taking the logs) is constant everywhere except when a shock occurs, because only in such cases would the output gap be zero in the absence of shocks (or if the shock lies infinitely far in the past): in the absence of shocks, $E_t(p_{t+1}) = p_{t+1}$ and equation (II.6) becomes $\Delta\pi_{t+1} = \frac{\alpha\lambda^2}{1-\lambda}(p_t - m_t)$, which yields $p_t = m_t$ only if m_t is an affine function of time. Let's thus consider that money-supply dynamics is given by $m_t = m_{old,t} = m_{-1} + (t+1) g_{old}$ until a shock occurs at $t = 0$ such that $m_t = m_{-1} + \delta + (t+1) g$ for $t \geq 0$. Then, the solution is:

$$\begin{aligned} p_t &= m_t - \theta^{t+1}\delta \text{ for } t \geq 0 \\ p_t &= m_{old,t} \text{ for } t < 0. \end{aligned} \tag{II.9}$$

Thus, adjustment is perfect and immediate if $\delta = 0$ (even if there is a change in the growth rate).

Strategic neutrality

In the case of strategic neutrality ($\alpha = 1$), equation (II.2) becomes:

$$p_t = (1-\lambda)\, p_{t-1} + \lambda^2 \gamma \sum_{k=0}^{\infty} (1-\gamma)^k \left[\sum_{j=0}^{\infty} \left[(1-\lambda)^j\, E_{t-k}\left(m_{t+j}\right) \right] \right]. \quad \text{(II.10)}$$

All future aggregate prices have disappeared (as in the sticky-information model), but the last-period aggregate price is still present: this is a first-order recursive equation with constant coefficients that can be solved easily:

$$p_t = \lambda^2 \gamma \sum_{k=0}^{\infty} (1-\gamma)^k \left[\sum_{l=0}^{\infty} \sum_{j=0}^{\infty} \left[(1-\lambda)^{j+l}\, E_{t-l-k}\left(m_{t-l+j}\right) \right] \right]. \quad \text{(II.11)}$$

Let's assume that there is only one shock, that it is not anticipated and that it occurs at $t = 0$. As above, $m_{old,t}$ is the actual money-supply before the shock or the money supply that would have prevailed after time $t = 0$ if no shock had occurred. Then $E_{t-l-k}\left(m_{t-l+j}\right) = m_{old,t-l+j}$ except if $t - k \geq l$ and $t + j \geq l$ (that is, except if $t + j \geq l$) in which case it is equal to m_{t-l+j}. Then equation (II.11) becomes for $t \geq 0$:

$$p_t = \lambda^2 \gamma \left\{ \begin{array}{c} \sum_{k=0}^{t} (1-\gamma)^k \left[\sum_{j=0}^{\infty} \sum_{l=0}^{t-k} \left[(1-\lambda)^{j+l} \left(m_{t-l+j} - m_{old,t-l+j} \right) \right] \right] \\ + \sum_{k=0}^{\infty} (1-\gamma)^k \left[\sum_{j=0}^{\infty} \sum_{l=0}^{\infty} \left[(1-\lambda)^{j+l}\, m_{old,t-l+j} \right] \right] \end{array} \right\}. \quad \text{(II.12)}$$

Under which conditions is the output gap zero in the absence of a shock? As shown above, this is the case for the sticky-price model only if monetary supply is an affine function of time. Since the strategic-neutrality case overlaps with the sticky-price case, being an affine function of time is a necessary condition for money-supply dynamics to yield a zero output gap in the absence of a shock for every parameter value of the strategic-neutrality case. That this necessary condition is also sufficient is most easily seen in the Phillips curve (4.1) itself.

- **Permanent shock to the money-supply level and growth rate**

  Let's focus, as in the sticky-price model, on money-supply dynamics given by $m_t = m_{old,t} = m_{-1} + (t+1)\, g_{old}$ until an unanticipated shock occurs at $t = 0$ such that $m_t = m_{new,t} = m_{-1} + \delta + (t+1)\, g$ for $t \geq 0$. Then, for $t \geq 0$, the solution is:[27]

$$p_t = \begin{array}{c} m_t + \delta \left[\frac{\lambda}{\gamma-\lambda} (1-\gamma)^{t+2} - \frac{\gamma}{\gamma-\lambda} (1-\lambda)^{t+2} \right] \\ + (g - g_{old}) \left[\begin{array}{c} (t+1)(1-\gamma)^{t+2} \frac{\lambda}{\gamma-\lambda} \\ + \gamma \frac{(1-\gamma)(1-\lambda)\left[(1-\gamma)^{t+1} - (1-\lambda)^{t+1}\right]}{(\gamma-\lambda)^2} \end{array} \right] \end{array} \text{ for } \lambda \neq \gamma$$

$$\text{(II.13)}$$

$$p_t = m_t - \left\{ \begin{array}{c} \delta (1-\lambda)^{t+1} \left[1 + (t+1)\lambda\right] \\ + (g - g_{old})(t+1)(1-\lambda)^{t+1} \left(1 + \frac{\lambda}{2} t\right) \end{array} \right\} \text{ for } \lambda = \gamma.$$

  From this equation, inflation could be computed. In particular, if the monetary shock occurs only to the level of money supply (that is, $g = g_{old}$), then inflation is simply given (for $t \geq 0$) by $\pi_t - g = \delta\, \Xi_t$, where Ξ_t is the probability that a firm sets its first informed price-adjustment t periods after the shock.

- **Persistent but not permanent shocks to the money-supply growth rate**

  Let's consider the following shock:

  $m_{old,t} = m_{-1} + (t+1)\, g_{old}$ for $t < 0$

  $$m_t = m_{-1} + (t+1)\, g_{old} + \varepsilon_0 \sum_{i=0}^{t} \rho^i \text{ for } t \geq 0.$$

  This implies that $\Delta m_t - g_{old} = \rho\left(\Delta m_{t-1} - g_{old}\right) + \varepsilon_t$, where $\varepsilon_t = \varepsilon_0$ if $t = 0$ and zero otherwise.

  Thus, ρ is the autocorrelation of the money-supply growth rate. If $\rho = 0$, then the shock is completely transitory (or equivalently, it is a permanent shock to the level of money supply). If $\rho = 1$, then this is a permanent shock to the money-supply growth rate. Then, for $t \geq 0$, the solution is:

[27] If there is a $m_{new,t}$ instead of a m_t in (II.13), then this formula is also valid for $t = -1$ (with $m_{new,t=-1} = m_{-1} + \delta$).

a) for $\lambda \neq \gamma$

$$p_t = m_t + \frac{1}{1-\rho}\varepsilon_0 \left\{ \begin{array}{c} \frac{\lambda}{\gamma-\lambda}(1-\gamma)^{t+2} - \frac{\gamma}{\gamma-\lambda}(1-\lambda)^{t+2} \\ +\rho^{t+1} \\ +\frac{\rho}{1-\frac{1-\rho}{\lambda}} \frac{1}{\rho+\frac{1-\rho}{\lambda}} \frac{1}{1-\lambda-\rho(1-\gamma)} \left[\begin{array}{c} \gamma(1-\lambda)^{t+2} \\ +(1-\lambda)\rho^{t+1}(1-\gamma)^{t+2} \\ -(1-\lambda)\rho^{t+1} \\ +\rho^{t+2}(1-\gamma) \left[\begin{array}{c} 1 \\ -(1-\gamma)^{t+1} \end{array} \right] \end{array} \right] \end{array} \right\}. \quad \text{(II.14)}$$

b) for $\lambda = \gamma$

$$p_t = m_t + \varepsilon_0 \frac{1}{1-\frac{1-\rho}{\lambda}} \frac{1}{1+(1-\rho)\frac{1-\lambda}{\lambda}} \left\{ \begin{array}{c} -\frac{1-\rho^{t+1}}{1-\rho}\rho(1-\lambda)^{t+1} \\ +\lambda(1-\lambda)^{t+1}\left(\frac{1-\rho^{t+1}}{1-\rho} - (t+1)\right)\frac{\rho}{1-\rho} \\ +\frac{1-\lambda}{\lambda^2}(1-\rho)\left[(1-\lambda)^{t+1} + (1-\lambda)^{t+1}\lambda(t+1) - \rho^{t+1}\right] \end{array} \right\}.$$

Here again, the inflation dynamics can be computed.